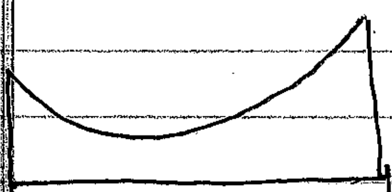
5.1 Numerical Sums

We can use rectangles to <u>approximate</u> the area under a curve.

Ex 1 Approximate the area under $f(x) = x^2 + 1$ on $[-2, 5]$ using $N = 7$ rectangles (right endpoints)



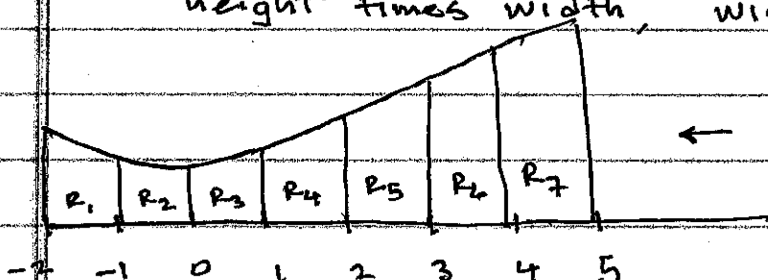
1) Build the partition:

$$h = \frac{5-(-2)}{7} = 1$$

$x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 2$, $x_5 = 3$, $x_6 = 4$, $x_7 = 5$

2) Find the area of each rectangle:

height times width, width $= h = 1$



← what are the heights?

Use the right endpoint, so R_1 height is $f(-1)$

R_2 $f(0)$

etc

R_7 $f(5)$

$R_1 = h * f(-1) = 1 * ((-1)^2 + 1) = 2$

$R_2 = h * f(0) = 1 * (0^2 + 1) = 1$

$R_3 = h * f(1) = 1 * (1^2 + 1) = 2$

$R_4 = h * f(2) = 1 * (2^2 + 1) = 5$

$R_5 = h * f(3) = 1 * (3^2 + 1) = 10$

$R_6 = h * f(4) = 1 * (4^2 + 1) = 17$

$R_7 = h * f(5) = 1 * (5^2 + 1) = 26$

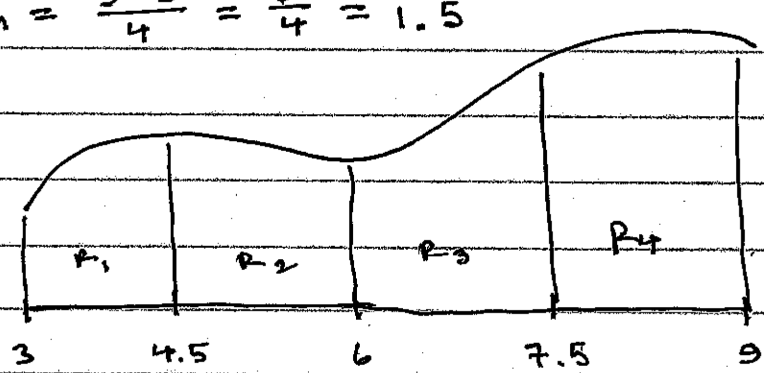
3) Add the rectangle areas for the total estimate

$$A = \sum_1^7 R_i = 2+1+2+5+10+17+26 = 63$$

(for reference, the actual area is 51.3; using more rectangles will give a better approximation

Ex 2 Approximate the area under $f(x) = \sin x + x$ on $[3, 9]$ using $N = 4$ rectangles

$$h = \frac{9-3}{4} = \frac{6}{4} = 1.5$$



Numerical Sums 3

Again, use the right endpoint for the height

$R_1 = h * f(4.5) = 1.5 * (\sin(4.5) + 4.5) = 5.28$

radian mode

$R_2 = h * f(6) = 1.5 * (\sin(6) + 6) = 8.58$

$R_3 = h * f(7.5) = 1.5 * (\sin(7.5) + 7.5) = 12.66$

$R_4 = h * f(9) = 1.5 * (\sin(9) + 9) = 14.12$

total area $A \approx R_1 + R_2 + R_3 + R_4 = 40.64$

(actual area is 35.92)