4.9 Practice Problems

1 a $f'(x) = \frac{3}{8}x^{1/3} + \frac{7}{11}x^{-1/7}$

$f(x) = \frac{3}{8}\left(\frac{x^{4/3}}{4/3}\right) + \frac{7}{11}\left(\frac{x^{6/7}}{6/7}\right) + C$

$= \frac{9}{32}x^{4/3} + \frac{49}{66}x^{6/7} + C$

b $f'(x) = \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{2\sqrt{x}} + \frac{1}{2}\sqrt{x}$

$= x^{-1/2} + x^{1/2} + \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{1/2}$

$f(x) = \frac{x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} + \frac{1}{2}\cdot\frac{x^{1/2}}{1/2} + \frac{1}{2}\frac{x^{3/2}}{3/2} + C$

$= 2\sqrt{x} + \frac{2}{3}x^{3/2} + \sqrt{x} + \frac{1}{3}x^{3/2} + C$

c $f'(x) = 6\cos x - 3\sin x$ note $(\cos x)' = -\sin x$

$f'(x) = 6\sin x + 3\cos x + C$ so $(-\cos x)' = \sin x$

d $f'(x) = 5\sec^2 x + 2\sec x\tan x$

$f(x) = 5\tan x + 2\sec x + C$

e $f'(x) = 3\csc^2 x + 4\csc x\cot x$ $(\cot x)' = -\csc^2 x$

$f(x) = -3\cot x - 4\csc x + C$ so $(-\cot x)' = \csc^2 x$

f $f'(x) = 2\sinh x + 7\cosh x$

$f(x) = 2\cosh x + 7\sinh x + C$

2

1g $f'(x) = \frac{1}{x} + \frac{1}{x^2} + e^x + \frac{1}{3x} - 2e^x$

↑ ln ↑ power

$= \frac{1}{x} + x^{-2} + e^x + \frac{1}{3}\left(\frac{1}{x}\right) - 2e^x$

$f(x) = \ln x + \frac{x^{-1}}{-1} + e^x + \frac{1}{3}\ln x - 2e^x$

↖ $-\frac{1}{x}$

h $f'(x) = \frac{6}{1+x^2} + \frac{2}{\sqrt{1-x^2}} + \frac{1}{\sqrt{4-4x^2}}$

$\frac{1}{\sqrt{4(1-x^2)}} = \frac{1}{2\sqrt{1-x^2}}$

$f(x) = 6\tan^{-1}x + 2\sin^{-1}x + \frac{1}{2}\sin^{-1}x$

2a $f'(x) = 3x^4 - 2x + e^x \qquad f(0) = 3$

$f(x) = \frac{3}{5}x^5 - x^2 + e^x + c$

$3 = f(0) = 0 - 0 + 1 + c \qquad c = 2$

$f(x) = \frac{3}{5}x^5 - x^2 + e^x + 2$

b $f'(x) = \cos x + \sin x \qquad f(0) = 8$

$f(x) = \sin x - \cos x + c$

$8 = f(0) = 0 - 1 + c \qquad c = 9$

$f(x) = \sin x - \cos x + 9$

3

3 model rocket. initial position $s(0) = 5$

" velocity $s'(0) = 75$

(up is positive)

$g = 10$

$s''(t) = -10$

$s'(t) = -10t + c$

$75 = s'(0) = -0 + c, \quad c = 75$

$s'(t) = -10t + 75$

$s(t) = -10\left(\frac{t^2}{2}\right) + 75t + d = -5t^2 + 75t + d$

$5 = s(0) = -0 + 0 + d \quad d = 5$

$s(t) = -5t^2 + 75t + 5$

4 textbook. $s(0) = 50 \quad s'(0) = -10$

(down is negative)

units are feet so use $g = 32 \text{ ft/s}^2$

$s''(t) = -32$

$s'(t) = -32t + c = -32t - 10$

$s(t) = -16t^2 - 10t + d = -16t^2 - 10t + 50$

hitting the ground means $s(t) = 0$

4

$0 = -16t^2 - 10t + 50$

$$t = \frac{10 \pm \sqrt{100 - 4(-16)50}}{2(-16)} = \frac{10 \pm \sqrt{3300}}{-32}$$

$$= -2.1, 1.48$$

throw out the negative root; it takes almost 1.5 seconds for the textbook to hit the ground