4.5 Typical Problems

A Given $f(x)$, find $f'(x)$, $f''(x)$, build the sign chart and sketch the graph.

$f(x)$ – used to find roots, VAs, end behavior
$f'(x)$ – gives CPs, inc/dec intervals
$f''(x)$ – gives IPs, concavity

Ex Sketch $f(x) = \dfrac{1-x^2}{x^3}$

1 • roots, $f(x) = 0$ where $1-x^2 = 0$, so $x = \pm 1$.
2 • VA, where denominator $= 0$, so $x = 0$
3 • end behavior: $f(x) \approx \dfrac{-x^2}{x^3} = -\dfrac{1}{x}$

as $x \to \pm\infty$, $-\frac{1}{x} \to 0$, so graph has a horizontal asymptote $y = 0$ at both ends

• $\cancel{f'(x) = -3x^{4}}$ write $f(x) = \dfrac{1}{x^3} - \dfrac{x^2}{x^3} = x^{-3} - x^{-1}$
so $f'(x) = -3x^{-4} + x^{-2} = -\dfrac{3}{x^4} + \dfrac{1}{x^2}$
$= \dfrac{-3+x^2}{x^4}$.

4 CP: $f'(x)$ DNE at $x = 0$ (denominator). This location is already a VA, so no surprize
$f'(x) = 0$ where $-3+x^2 = 0$, so $x = \pm\sqrt{3}$

CP: $x = -\sqrt{3}, +\sqrt{3}$

4.5 p 2

$f' = -3x^{-4} + x^{-2}$ so

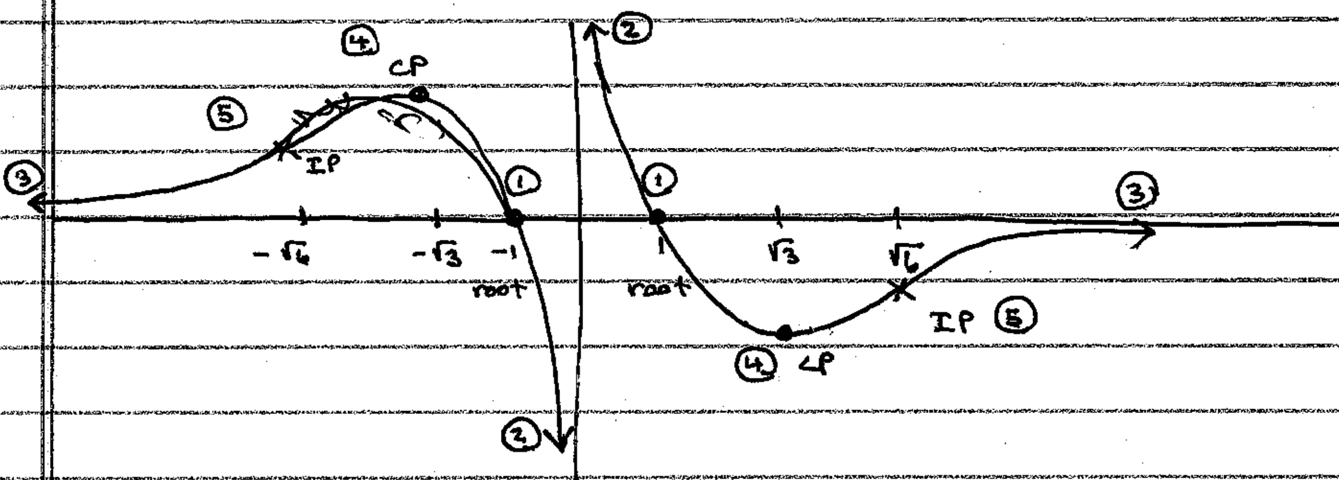
$f'' = +12x^{-5} - 2x^{-3} = \frac{12}{x^5} - \frac{2}{x^3} = \frac{12-2x^2}{x^5}$

(5) $f'' = 0$ where $12 - 2x^2 = 0$ so $x^2 = 6$, $x = \pm\sqrt{6}$

these are PIPs: possible inflection points

(6) use the VA, CP and PIP to build a sign chart

		PIP $-\sqrt{6}$		CP $-\sqrt{3}$		VA 0		CP $\sqrt{3}$		PIP $\sqrt{6}$	
f'	+		+		−		−		+		+
f''	+		−		−		+		+		−



B Sketch a graph given its properties

Ex Sketch $f(x)$ if

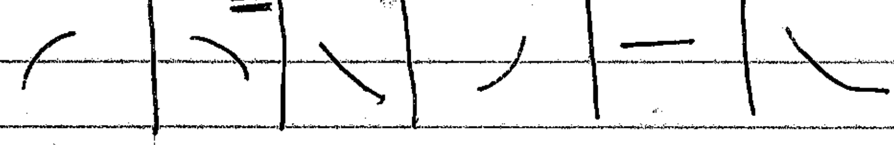
- $f(x)$ is continuous
- $f(-2)=5,\ f(0)=1,\ f(3)=4$
- $f'>0$ on $(-\infty,-2)\cup(0,3)$, $f'<0$ on $(-2,0)\cup(5,\infty)$
- $f'=0$ on $(3,5)$
- $f'(3),\ f'(5)$ DNE
- $f''<0$ on $(-\infty,-1)$, $f''>0$ on $(-1,3)\cup(5,\infty)$, $f''=0$ on $(3,5)$
- $\lim\limits_{x\to-\infty} f(x)=-\infty$, $\lim\limits_{x\to\infty} f(x)=0$

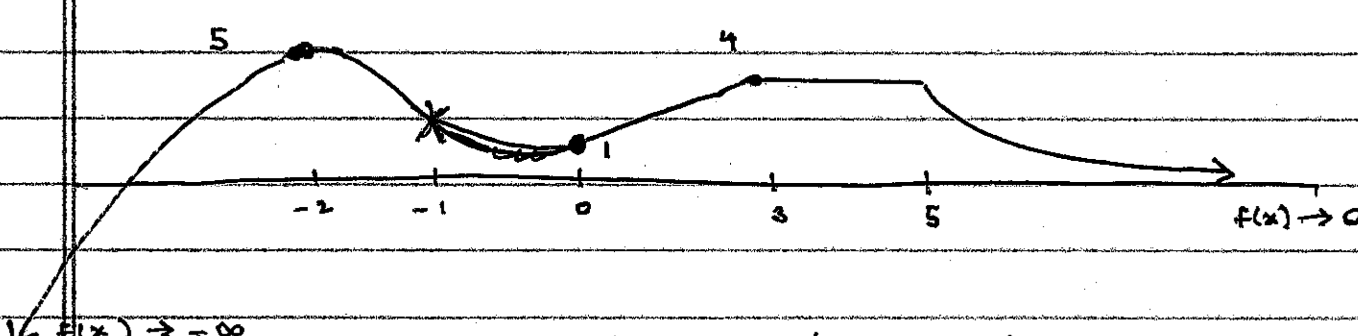
① Gather all the special x values mentioned:

$x=-2,0,3,5,-1$

② Build the sign chart

		-2		-1		0		3		5	
f'	$+$		$-$		$-$		$+$		0		$-$
f''	$-$		$-$	IP	$+$		$+$		0		$+$





Notice that I can't draw either.

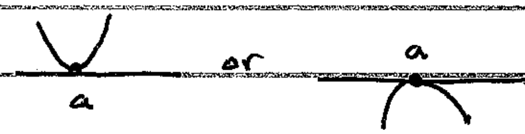
Mark CPs with •, IPs with x.

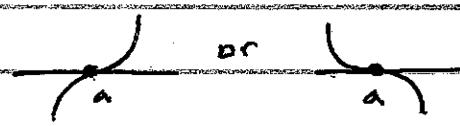
4.5 p 4

C Use roots and end behavior to draw a quick sketch.

This works only for factored polynomials

Remember, a single root $(x-a)$ looks like 

A double root $(x-a)^2$ looks like 

A triple root $(x-a)^3$ looks like 

For end behavior, look at the sign of each factor to tell whether the function goes to $+\infty$ or $-\infty$

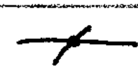
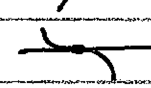
Ex Sketch $f(x) = x^2(2-x)(x-3)^3$

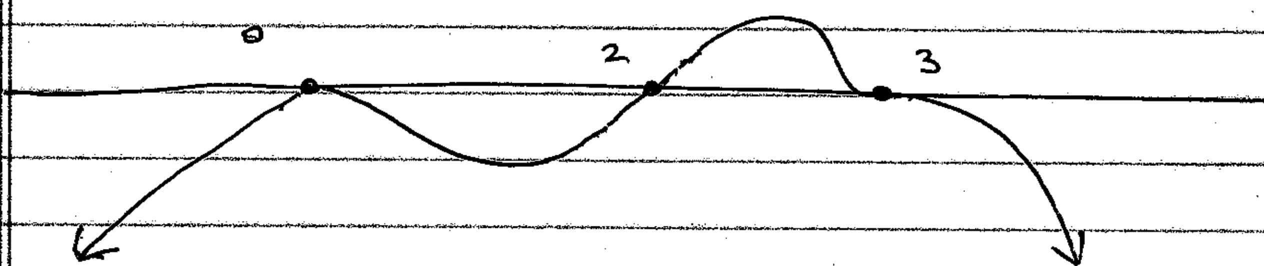
roots: $x = 0, 2, 3$

as $x \to +\infty$, $f(x)$: $(+)^2(-)(+)^3$: $-\infty$

as $x \to -\infty$, $f(x)$: $(-)^2(+)(-)^3$: $-\infty$

note: since f is a polynomial, you know f goes to $\pm\infty$. Use the signs to determine which.

- Start the sketch on the left. It comes up from $-\infty$
- Root at $x=0$ must look like  (the x^2)
- Root at $x=2$ " " "  $2-x$
- Root at $x=3$ " " "  $(x-3)^3$
- Then it's off to $-\infty$



We're not tracking the critical points, inflection points, or max/min values

We do a little bit of work and get a little bit of information.

If we want that other information, we need f', f'' and the sign chart — that's a problem in the type A category.