Section 4.4D

Limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

These are already set up to apply L'H, so let's see how it works out

Ex 8 $\quad L = \lim_{x \to 0} \frac{e^x - 1}{x} \qquad \left(\frac{1-1}{0} = \frac{0}{0} \right)$

$\overset{L'H}{=} \lim_{x \to 0} \frac{e^x}{1} = e^0 = 1$

Ex 9 $\quad L = \lim_{x \to 0} \frac{\sin x}{\ln(x+1)} \qquad \left(\frac{\sin 0}{\ln 1} = \frac{0}{0} \right)$

$\overset{L'H}{=} \lim_{x \to 0} \frac{\cos x}{\frac{1}{x+1}} = \frac{1}{\frac{1}{0+1}} = \frac{1}{1} = 1$

- Rational functions don't need L'H

Ex 10 $\quad L = \lim_{x \to \infty} \frac{2x^2+3}{3x^3+7} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \infty} \frac{2 + 3/x^2}{3\underline{x} + 7/x^2}$

$= \frac{2+0}{\underline{\infty} + 0} = 0$

Ex 11 $\quad L = \lim_{x \to \infty} \frac{2x^2+3}{3x^2+7} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \infty} \frac{2 + 3/x^2}{3 + 7/x^2} = \frac{2+0}{3+0} = \frac{2}{3}$

Ex 12 $\quad L = \lim_{x \to \infty} \frac{2x^3+3}{3x^2+7} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \infty} \frac{2x + 3/x^2}{3 + 7/x^2} = \frac{\infty + 0}{3+0} = \infty$

Ex 13 $L = \lim_{x\to\infty} \dfrac{e^x}{x^2} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{e^x}{2x} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{e^x}{2} = \dfrac{\infty}{2} = \infty$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

Ex 14 $L = \lim_{x\to\infty} \dfrac{e^x}{\ln x} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{e^x}{1/x} = \dfrac{\infty}{1/\infty} = \dfrac{\infty}{0} = \infty$

$\frac{\infty}{\infty}$

Ex 15 $L = \lim_{x\to\infty} \dfrac{\ln x}{x^n} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{1/x}{n x^{n-1}} = \dfrac{1/\infty}{\infty} = \dfrac{0}{\infty} = 0$

$n \geq 1$ $\frac{\infty}{\infty}$

- Notice in 13, 14 the e^x dominates, so $e^x \to \infty$ faster than $x^2 \to \infty$ and $\ln x \to \infty$. In 13, if we had x^{157}, we could apply L'H 157 times. But it's better to note that $\lim_{x\to\infty} \dfrac{e^x}{x^p} = \infty$ for <u>any</u> $p > 0$

In 15, the x^n dominates $\ln x$, so $x^n \to \infty$ faster than $\ln x$. We can write this as a "heirarchy" used in ch 11 in Calc II

$$\ln x \ll x^p \ll e^x \quad \text{as } x \to \infty$$

$\ll$ reads "much less than", so $\ln x \ll x^p$ is the mathematical expression for "$x^p \to \infty$ faster than $\ln x \to \infty$"

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Graphically