Section 4.4 C

Limits of the Form $0 \cdot \infty$

Convert the product to a fraction. Whatever you move to the denominator becomes a reciprocal, so pick the factor that will be easier to differentiate. Symbolically, if $f(x) \to 0$, $g(x) \to \infty$

$$f(x)\,g(x) = \frac{f(x)}{1/g(x)} \to \frac{0}{1/\infty} = \frac{0}{0}, \text{ apply L'H}$$

or

$$= \frac{g(x)}{1/f(x)} \to \frac{\infty}{1/0} = \frac{\infty}{\infty}, \text{ apply L'H}$$

Ex 5 $\quad L = \lim_{x\to\infty} e^{-x} \cdot \ln x \qquad e^{-x} \to 0,\ \ln x \to \infty,$ so $0 \cdot \infty$

implied fraction

$$L = \lim_{x\to\infty} \frac{\ln x}{e^x} \qquad \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{1/x}{e^x} = \frac{1/\infty}{\infty} = \frac{0}{\infty} = 0$$

note: $\frac{\ln x}{e^x} \to 0$ implies $e^x \to \infty$ faster than $\ln x \to \infty$

Ex 6 $\quad L = \lim_{x\to\infty} x\left(\tan^{-1}x - \frac{\pi}{2}\right) \qquad \infty \cdot \left(\frac{\pi}{2} - \frac{\pi}{2}\right)$ so $\infty \cdot 0$

which do you prefer: $\dfrac{\tan^{-1}x - \frac{\pi}{2}}{1/x}$ or $\dfrac{x}{1/(\tan^{-1}x - \frac{\pi}{2})}$

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so... $L = \lim_{x \to \infty} \dfrac{\tan^{-1} x - \frac{\pi}{2}}{1/x}$ $\qquad \dfrac{\frac{\pi}{2} - \frac{\pi}{2}}{1/\infty} = \frac{0}{0}$

$\overset{L'H}{=} \lim_{x \to \infty} \dfrac{\frac{1}{x^2+1}}{-1/x^2}$ $\qquad \leftarrow \dfrac{1}{-1/x^2} = -x^2$

$= \lim_{x \to \infty} \dfrac{-x^2}{x^2+1}$ $\qquad \frac{\infty}{\infty}$, needs more work

(i) divide top and bottom by x^2

(ii) apply L'H twice

(i) $L = \lim_{x \to \infty} \dfrac{-x^2}{x^2+1} \cdot \dfrac{1/x^2}{1/x^2} = \lim_{x \to \infty} \dfrac{-1}{1+1/x^2} = \dfrac{-1}{1+0} = -1$

(ii) $L \overset{L'H}{=} \lim_{x \to \infty} \dfrac{-2x}{2x+0} \overset{L'H}{=} \lim_{x \to \infty} \dfrac{-2}{2} = -1$

Ex 7 $\quad L = \lim_{x \to \infty} x \ln x = \infty \cdot \infty = \infty$

determinate from the start