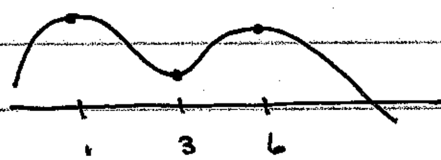
4.3 Shape of Graphs

$1^{ST}$ derivative

if $f'(x) > 0$ on $(a, b)$, $f$ is increasing

$<$ decreasing

Ex

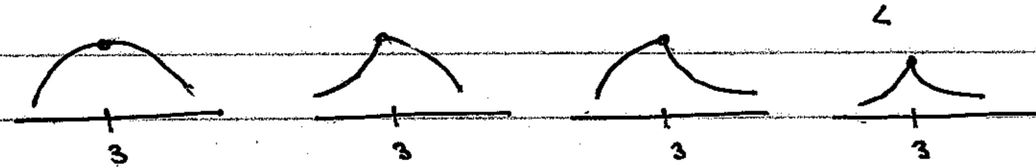


inc on $(-\infty, 1) \cup (3, 6)$

dec $(1, 3) \cup (6, \infty)$

Ex Sketch a fcn $f(x)$ for which $f' > 0$ on $(-\infty, 3)$

$<$ $(3, \infty)$

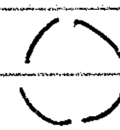


First Derivative Test

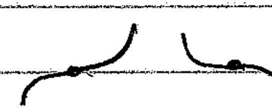
Let $x = c$ be a CP of $f(x)$. Then

$f$ has a local max if $f'$ changes from $+$ to $-$

min $-$ $+$



Saddle point if $f'$ dn change sign



Ex Find where $f(x) = x^4 - 4x^3 - 2x^2 + 12x$ is inc/dec

Find CP : $f' = 4x^3 - 12x^2 - 4x + 12$

$= 4(x^3 - 3x^2 - x + 3)$

$= 4(x^2(x-3) - (x-3))$

$= 4(x^2 - 1)(x - 3)$

CP $x = -1, 1, 3$

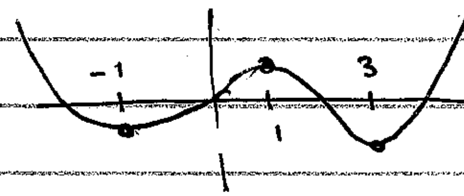
4.3 2

sign chart

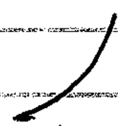
| | $-1$ | $1$ | $3$ | |
|---|---|---|---|---|
| $(x-3)$ | $-$ | $-$ | $-$ | $+$ |
| $(x^2-1)$ | $+$ | $-$ | $+$ | $+$ |
| $f'(x)$ | $-$ | $+$ | $-$ | $+$ |

inc on $(-1,1) \cup (3,\infty)$

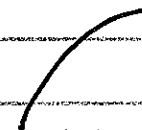
dec on $(-\infty,-1) \cup (1,3)$




$2^{nd}$ derivative - concavity




vs

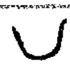



$f'$ is inc so — $f'$ is dec so

its deriv is + — its deriv is −

$f'' > 0$ — $f'' < 0$

concave up CUP — concave down CAP

think $y = x^2$  — $y = -x^2$ 

$y'' = 2$ — $y'' = -2$

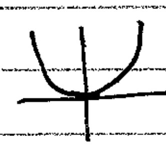
Inflection point: a point on graph where concavity changes

IP

Often, $f'' = 0$ but not always

$f = x^4$ — $f''(0) = 0$ but

$f'' = 12x^2$ — conc dn $\Delta$



4.3 3

SECOND DERIVATIVE TEST . Let $x=c$ be a CP of $f$

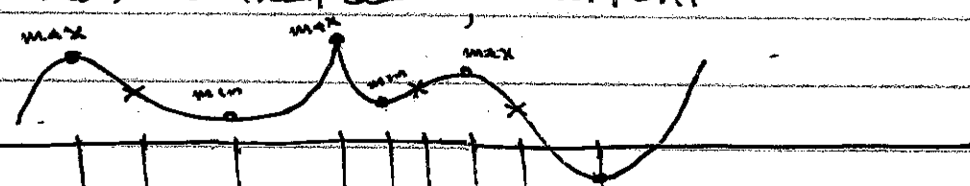
If $f''(c) > 0$ then $f$ has a loc min at $x=c$ ∪

$<$ max ∩

$f''(c) = 0$, you don't know what's happening

Ex Find where $f(x)$ is inc/dec, CUP/CAP

graphical




| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | CP | | CP | CP | | | CP | | CP | |
| $f'$ | + | − | − | + | − | + | + | − | − | + |
| $f''$ | − | − | + | + | + | + | − | − | + | + |
| | | IP | | not an IP | | IP | | IP | | |

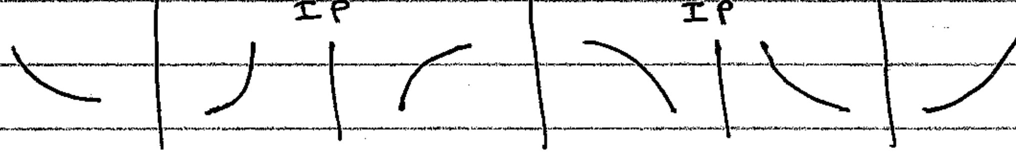
analytical

$f(x) = (x^2-1)^2$

$f'(x) = 2(x^2-1)(2x) = 4x^3 - 4x = 4x(x^2-1)$ CP $x = -1, 0, 1$

$f''(x) = 12x^2 - 4 = 4(3x^2-1)$ $= 0$ at $x = \pm\frac{1}{\sqrt{3}}$ PIP

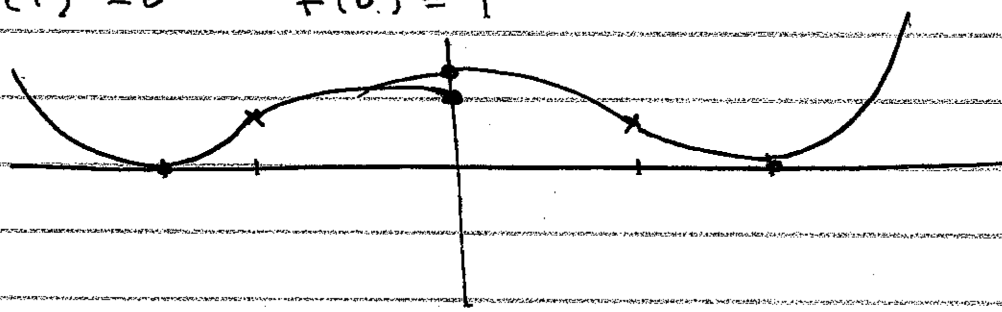
| | | CP $-1$ | | $-\frac{1}{\sqrt{3}}$ | | CP $0$ | | $\frac{1}{\sqrt{3}}$ | | CP $1$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | − | | − | | − | | + | | + | | + |
| $x^2-1$ | + | | − | | − | | − | | − | | + |
| $f'$ | − | min | + | | + | max | − | | − | min | + |
| $f''$ | + | | + | IP | − | | − | IP | + | | + |



4.3 4

$f(-1) = 0 \quad f\left(\pm \frac{1}{\sqrt{3}}\right) = \left(\frac{1}{3} - 1\right)^2 = \frac{4}{9}$

$f(1) = 0 \quad f(0) = 1$



Ex Sketch f if $f(0) = 0 \quad f(-1) = 1 \quad f'(-1) = 0$

$f' > 0$ for $x > 0$

$f'' > 0 \ (-\infty, -1) \quad f'' < 0$ on $(-1, 0) \cup (0, \infty)$

~~$f''(0)$ DNE~~

| | $-1$ | | $0$ | |
|---|---|---|---|---|
| $f'$ | ? | ? | + |
| $f''$ | + | $-$ | $-$ |