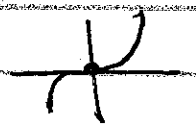
4.2 MVT

- If $f(x)$ has a local max/min at $x=c$, then
  & $x=c$ is a CP ($f'(c)=0$ or DNE)

But ... ~~if $f'=0$~~ not all points where $f'(c)=0$ or DNE lead to a local max/min

Ex $f(x)=x^3$  $f'(0)=0$ saddle point

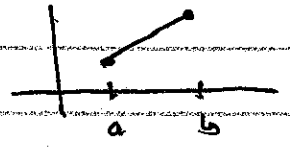
$f(x)=x^{1/3}$  $f'(x)=\frac{1}{3}x^{-2/3}$ $f'(0)$ DNE

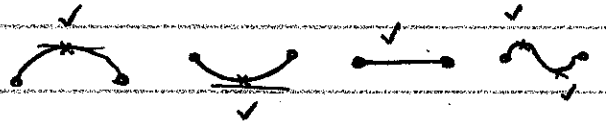
vertical tangent

Question. What conditions on $f(x)$ force the existence of CP of the type $f'(c)=0$ & for $x\in[a,b]$?

1) f must be continuous and differentiable at all x so $f'(x)$ exists. Is that enough?

NO :



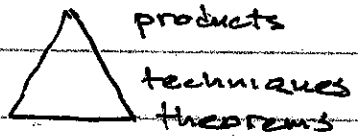
2) Let's require $f(a)=f(b)$ 

Careful about (1) f must be cont on $[a,b]$ to avoid 

(2) $f'(x)$ doesn't need to exist at $x=a$ or b 

Rolle's Thm. Let f be cont on $[a,b]$ and diff on (a,b) with $f(a)=f(b)$. There is a point $c\in(a,b)$ where $f'(c)=0$.

Math'l tool w/ no direct utility



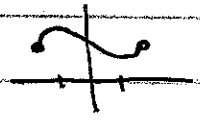
Ex Show $f(x) = x^3 - x + 1$ satisfies Rolle's Thm on $[-1, 1]$

- f is cont + diff since it's a poly
- $f(-1) = 1 = f(1)$

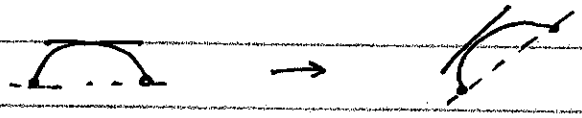
so $\exists\ c \in (-1,1)$ where $f'(c) = 0$

Finding c = direct computation

$$0 = f'(x) = 3x^2 - 1 \rightarrow x = \pm \frac{1}{\sqrt{3}}$$



What can we say if $f(a) \neq f(b)$?



Mean Value Thm

Let $f(x)$ be cont on $[a,b]$ and diff on (a,b). There is a $c \in (a,b)$ where $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

tan line — secant line

Ex. Apply the MVT to $f(x) = 2x^3 + x^2 - x - 1$ on $[0, 2]$

- f is cont + diff bc it's a poly
- $f(0) = -1$, $f(2) = 17$
- $\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{18}{2} = 9$. There is a point c in $(0,2)$ where $f'(c) = 9$

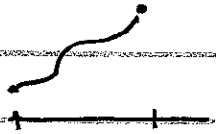
Ex Apply MVT to $f(x) = \frac{x+1}{x-1}$ on $[0, 2]$

MVT would say $f'(c) = \dfrac{f(2) - f(0)}{2 - 0} = \dfrac{3 - (-1)}{2} = 2$

but $f'(x) = \dfrac{-2}{(x-1)^2} < 0$ can't!

Ex At 10:05 AM Harry passes mile marker 14. At 11:25am, he passes MM 119. Should he get a speeding ticket?

time position cont + diff

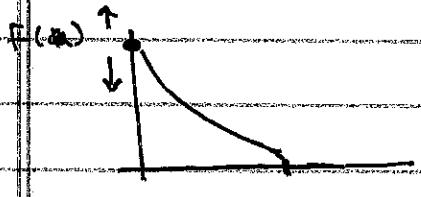


$a = 0$ $f(a) = 14$

$b = 1.33$ hrs $f(b) = 119$ $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$= \dfrac{119 - 14}{1.33 - 0} = 78.95$ mph. At some instant, Harry was going that fast. Nail him!

Ex Build a slide that has slope -4 at one point



$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$-4 = \dfrac{0 - f(a)}{10 - 0}$

$a = 0$ $f(a) = ?$

$b = 10$ $f(b) = 0$ $-40 = -f(a)$ so $f(a) = 40$