4.1 Max and Min Values

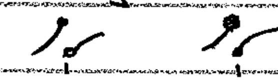
maximum — maxima

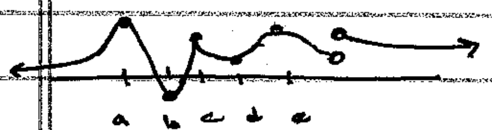
minimum — minima

Y values!

1) absolute max/min – highest/lowest point on <u>entire</u> graph

2) relative/local m/m – h/L point in a small neighborhood





f(a): ~~local~~ max and abs max

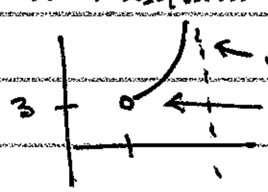
f(b): local min and abs min

f(c) local max

f(d) local min

f(e) local max

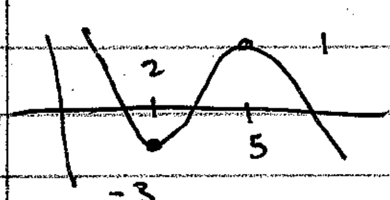
A max/min point must be <u>on</u> the graph



f(x) never equals 3

A local max/min occurs at a point $x=c$
where $f'(c)=0$ or $f'(c)$ DNE (Fermat's Thm)

The x location of a max/min value is called a critical number or critical point.



local min: -3 at CP $x=2$

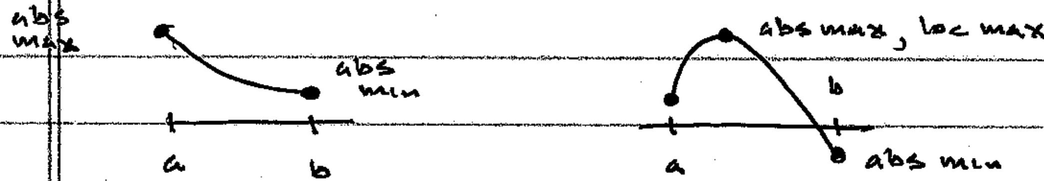
local max: 1 at CP $x=5$

4.1 2

- If we consider $y = f(x)$ restricted to $x \in [a, b]$

then abs max/min – at endpoint or in interior

local max/min – interior only



Types of problems

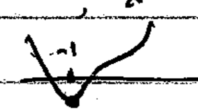
A Find the CP of $f(x)$

B Find the abs max/min of $f(x)$ on $[a, b]$ Closed Interval Method

Ex A Find the CP of $f(x)$

1. $f(x) = x^4 + 4x + 1$ CP $f'(x) = 0$ or DNE
not here

$f'(x) = 4x^3 + 4 = 4(x^3 + 1) = 0$: $x^3 = -1$, so $x = -1$



min $= f(-1) = 1 - 4 + 1 = -2$

2. $f(x) = x^3 - 2x^2 - 4x + 5$

$f'(x) = 3x^2 - 4x - 4 = (3x+2)(x-2)$

$f'(x) = 0$ at $x = -2/3, 2$ CP

f' DNE : not here

3. $f(x) = x\sqrt{6-x}$ domain $6 - x \geq 0$ $x \leq 6$

$f'(x) = \sqrt{6-x} + x\left(\frac{1}{2}\frac{1}{\sqrt{6-x}} \cdot (-1)\right)$

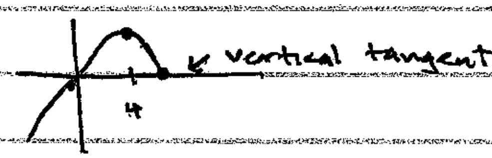
$= \sqrt{6-x} \cdot \frac{\sqrt{6-x}}{\sqrt{6-x}} - \frac{x}{2\sqrt{6-x}} = \frac{(6-x) - \frac{x}{2}}{\sqrt{6-x}} = \frac{6 - \frac{3}{2}x}{\sqrt{6-x}}$

4.1 3

$f'(x) = 0$ when num $= 0$ $\qquad 6 - \frac{3}{2}x = 0, \quad x = 4$

$f'(x)$ DNE when denom $= 0$ $\qquad x = 6$

CP are $x = 4, 6$



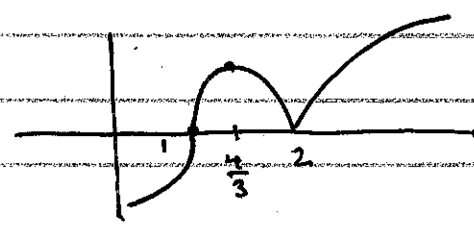
4. $f(x) = (x-1)^{1/3}(x-2)^{2/3}$ $\qquad$ domain $(-\infty, \infty)$

$$f'(x) = \left[\tfrac{1}{3}(x-1)^{-2/3}\right](x-2)^{2/3} + (x-1)^{1/3}\left[\tfrac{2}{3}(x-2)^{-1/3}\right]$$

$$= \frac{\frac{1}{3}(x-2)^{2/3}}{(x-1)^{2/3}} \cdot \frac{(x-2)^{1/3}}{(x-2)^{1/3}} + \frac{\frac{2}{3}(x-1)^{1/3}}{(x-2)^{1/3}} \cdot \frac{(x-1)^{2/3}}{(x-1)^{2/3}}$$

$$= \frac{\frac{1}{3}(x-2) + \frac{2}{3}(x-1)}{(x-1)^{2/3}(x-2)^{1/3}} = \frac{x - \frac{4}{3}}{(x-1)^{2/3}(x-2)^{1/3}}$$

CP : $x = \frac{4}{3}, 1, 2$



Ex B Find the abs max/min of $f(x) = x + \frac{1}{x}$ on $[.2, 4]$

location: CP or endpoint

CP : $f' = 1 - \frac{1}{x^2}$ $\qquad$ DNE at $x = 0$ (not in interval)

$= 0$ at $x = 1$

candidates

end	$x = .2$	$f(.2) = .2 + \frac{1}{.2} = 5.2$	← abs max
cp	$x = 1$	$f(1) = 1 + \frac{1}{1} = 2$	← abs min
end	$x = 4$	$f(4) = 4 + \frac{1}{4} = 4.25$	

Ex B2 Find the abs max/min of $f(x) = x^3 (x-4)^4$ on $[-1,3]$

$f'(x) = 3x^2 (x-4)^4 + 4x^3 (x-4)^3$

$= \ldots = x^2 (x-4)^3 (7x-12)$

cp $x = 0, 4, 12/7$

4: not in interval

candidates

$x = -1$ $f(-1) = -625$ ← abs min

$x = 0$ $f(0) = 0$

$x = 12/7$ $f(12/7) \approx 137.5$ ← abs max

$x = 3$ $f(3) = 27$