

Exam 3 Practice Problems – Calculus I

Below is a set of practice problems covering each of the topics on the study guide. It is not enough to work only these problems. You should also review your quiz problems, pertinent homework problems, and end of chapter review in the text, but focus only on those problems mentioned in the study guide.

1. Find the critical points of

(a) $f(x) = x^2e^x - 15e^x - 7$

(b) $g(x) = \frac{x^2}{x-3}$

(c) $h(x) = x^{1/2}(x+1)$

2. Find the absolute maximum and minimum values of the indicated function on the indicated interval

(a) $f(x) = -\frac{x^3}{3} + \frac{x^2}{2} + 6x + 1$ on $[0, 4]$

(b) $g(x) = \frac{x}{x^2 - x + 1}$ on $[0, 3]$

(c) $h(x) = e^x \cos x$ on $[0, 2\pi]$

3. Solve these application problems involving the Mean Value Theorem

(a) Betsy is driving along I77. At 1:00 pm, she is at mile marker 120. When she reaches mile marker 210, she notes that her average speed has been 66 mph. What time is it when she reaches mile marker 210?

(b) Your oven is at 70° when you turn it on. After 5 minutes, its temperature is 250° . Use the Mean Value Theorem to make a statement about the instantaneous rate of change of temperature (something like this: the instantaneous rate of change of temperature must equal xxx degrees/min at some time during that 5 minutes).

4. Find the intervals where the indicated function is increasing, decreasing, concave up and concave down.

(a) $f(x) = x^3 + 6x^2 + 9x - 1$

(b) $g(x) = \frac{x^2}{x+1}$

(c) $h(x) = x \ln x$

(d) $r(x) = \ln(x^2 + 9)$

(e) $s(x) = e^x \sin x$

5. Evaluate the following limits

(a) $L = \lim_{x \rightarrow \infty} (x^5 - x^4)$

(b) $L = \lim_{x \rightarrow \infty} x - e^x$

(c) $L = \lim_{x \rightarrow \infty} e^{-x} \tan^{-1} x$

(d) $L = \lim_{x \rightarrow -\infty} xe^x$

(e) $L = \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$

(f) $L = \lim_{x \rightarrow \infty} \frac{(\ln x)^3}{x^2}$

(g) $L = \lim_{x \rightarrow 1} \frac{x \ln x}{e^{x-1} - 1}$

(h) $L = \lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 6x}$

(i) $L = \lim_{x \rightarrow \infty} \frac{x^3 \ln x}{e^x}$

(j) $L = \lim_{x \rightarrow 0} x^{1/x}$

(k) $L = \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^{4x}$

(l) $L = \lim_{x \rightarrow 0} (1 + 5x)^{2/x}$

(m) $L = \lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^x$

6. Draw a sketch of a function that satisfies the given properties

(a) Sketch $f(x)$ if ... (a) $f(1) = 2$, $f(3) = 6$, (b) $f'(x) < 0$ on $(-\infty, 1) \cup (3, \infty)$ and $f'(x) > 0$ on $(1, 3)$, (c) $f''(x) < 0$ on $(2, 5)$ and $f''(x) > 0$ on $(-\infty, 2) \cup (5, \infty)$, (d) $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow 0$ as $x \rightarrow \infty$.

(b) Sketch $f(x)$ if ... (a) $f'(x) > 0$ on $(-\infty, 2)$ and $f'(x) < 0$ on $(2, \infty)$, (b) $f''(x) > 0$ on $(-\infty, 2) \cup (2, \infty)$, (c) $f(x) \rightarrow 2$ as $x \rightarrow -\infty$ and $f(x) \rightarrow 1$ as $x \rightarrow \infty$.

(c) Sketch $f(x)$ if ... (a) $f'(x) > 0$ on $(-\infty, 1) \cup (1, 3)$ and $f'(x) < 0$ on $(3, \infty)$, (b) $f''(x) > 0$ on $(1, 2)$ and $f''(x) < 0$ on $(-\infty, 1) \cup (2, \infty)$, (c) $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.

7. Sketch the following functions using only roots and end behavior

(a) $f(x) = x(x+1)^2(x-2)^2$

(b) $g(x) = (x+2)^4(x-1)$

(c) $h(x) = x^3(x-2)(x-3)$

8. Solve the following application problems

(a) You want to build a box with no top and a rectangular base (the length is twice the width). You have 10000 cm² of material to use for the sides and bottom. Find the dimensions of the box that maximize the volume.

(b) The universe has suddenly become two dimensional and the Earth is located at the point (0, 0) (yes, we're going medieval, the Earth is now the center of the universe). A comet moves along the path $3x - 4y = 9$, with units being millions of miles. How close does the comet come to Earth as it passes by?

(c) Some guy wants to build a cylindrical can using 500 cm² of material, and he insists on maximizing the volume. Find the radius and height of the can that will satisfy his demands. Hint: you will use both the volume and the surface area of a cylinder (the surface area includes top, bottom and sides).