Exam 2 Practice Problems

1a $f(x) = x(1+e^{4x})^{1/2}$

$f'(x) = (1+e^{4x})^{1/2} + x\left[\frac{1}{2}(1+e^{4x})^{-1/2} \cdot 4e^{4x}\right]$

b $f(x) = \dfrac{e^x}{1+xe^x}$

$f'(x) = \dfrac{(1+xe^x)e^x - e^x(e^x+xe^x)}{(1+xe^x)^2} = \dfrac{e^x - e^{2x}}{(1+xe^x)^2}$

c $f(x) = 6^{\ln x}$

i) recall $(b^y)' = [b^y \ln b]\,y'$ so $f' = [6^{\ln x} \cdot \ln 6]\,\frac{1}{x}$

ii) $6^{\ln x} = e^{\ln(6^{\ln x})} = e^{\ln x \cdot \ln 6}$ $\quad (e^{au})' = ae^{au}u'$

so $f' = e^{\ln x \cdot \ln 6} \cdot \ln 6 \cdot \frac{1}{x} = 6^{\ln x} \cdot \ln 6 \cdot \frac{1}{x}$

d $f(x) = \sec(\sinh e^x)$

$f'(x) = [\sec(\sinh e^x)\tan(\sinh e^x)][\cosh e^x][e^x]$

2 $f(x) = (1 + \sin x)^x$

$\ln f = \ln(1+\sin x)^x = x\ln(1+\sin x)$

$\frac{1}{f} f' = [1]\ln(1+\sin x) + x\left[\frac{1}{1+\sin x} \cdot \cos x\right]$

$f' = (1+\sin x)^x\left(\ln(1+\sin x) + \frac{x\cos x}{1+\sin x}\right)$

3 $f = x(1+\csc x)^3 \Big/ \sqrt{\ln(e^x+x)+x^3}$

$\ln f = \ln x + 3\ln(1+\csc x) - \frac{1}{2}\ln\left(\ln(e^x+x)+x^3\right)$

$\frac{1}{f}f' = \frac{1}{x} + 3\,\frac{1}{1+\csc x}\cdot(-\csc x\cot x)$

$\qquad - \frac{1}{2}\,\frac{1}{\ln(e^x+x)+x^3}\cdot\left(\frac{1}{e^x+x}\cdot(e^x+1) + 3x^2\right)$

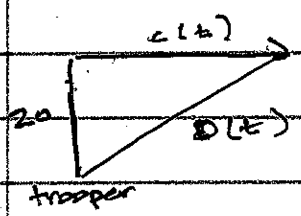
$f' = f\left[\qquad '' \qquad\right]$

4 $\sin(e^y) + x^4y^{-2} = \pi + e^x y^2$

$\cos(e^y)\cdot e^y\cdot y' + 4x^3y^{-2} - 2x^4y^{-3}y' = 0 + e^x y^2 + 2e^x y y'$

$y'\left(e^y\cos e^y - 2x^4y^{-3} - 2ye^x\right) = e^x y^2 - 4x^3y^{-2}$

$y' = \frac{''}{(\quad)}$

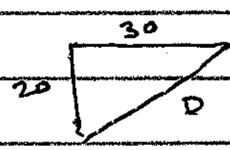
5 

given $\frac{dD}{dt} = 63$ mph

find $\frac{dc}{dt}$ when $c = 30$ ft

$c^2 + 20^2 = D^2 \qquad 2c\frac{dc}{dt} + 0 = 2D\frac{dD}{dt}$

$\frac{dc}{dt} = \frac{D}{c}\frac{dD}{dt}$ instant: $c = 30 \text{ ft} = \frac{30}{5280}$ mi



$D^2 = 20^2 + 30^2 = 1300$

$D = \sqrt{1300}$ ft $= \frac{\sqrt{1300}}{5280}$ mi

$\frac{dc}{dt} = \frac{\sqrt{1300}/5280}{30/5280}\cdot 63 \text{ mph} = \frac{\sqrt{1300}}{30}\cdot 63 \text{ mph}$

$= 75.7$ mph

6.  $V = \frac{4}{3}\pi r^3$ $\qquad \frac{dV}{dt} = 0.2$

Find $\frac{dr}{dt}$ when $r = 0.1$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$.2 = 4\pi \cdot .01 \cdot \frac{dr}{dt} \qquad \frac{dr}{dt} = \frac{.2}{4\pi \cdot .01} = 1.59 \text{ m/min}$$

7 $\quad 15.97^{3/2}$

$f(x) = x^{3/2} \qquad f'(x) = \frac{3}{2} x^{1/2} \qquad a = 16$

$f(16) = 64 \qquad f'(16) = 6$

$x^{3/2} \approx 64 + 6(x - 16)$

$15.97^{3/2} \approx 64 + 6(-.03) = 63.82$

8 $\quad e^{-.06}$

$f(x) = e^x \qquad f'(x) = e^x \qquad a = 0$

$f(0) = 1 \qquad f'(0) = 1$

$e^x \approx 1 + 1(x - 0)$

$e^{-.06} \approx 1 + (-.06) = .94$

9 $\quad V = \frac{4}{3}\pi r^3 \qquad r = 50 \pm 1.3 \text{ cm} \qquad |dr| \leq 1.3$

calculated volume $\quad V = \frac{4}{3}\pi(50)^3 = \frac{500{,}000}{3}\pi \text{ cm}^3$

$dV = 4\pi r^2\, dr$

$|dV| \leq 4\pi \cdot 50^2 \cdot 1.3 = 13{,}000\pi \text{ cm}^3$

relative error $\quad \frac{dV}{V} \leq \frac{13{,}000\pi}{\frac{500{,}000\pi}{3}}$

$$= \frac{39}{500} = .078 \quad (7.8\%)$$

10 $A = \pi r^2$ $\quad r = 50 \pm .8$ cm

calculated area $\quad A = \pi 50^2 = 2500\pi$

$dA = 2\pi r\, dr$

$|dA| \leq 2\pi \cdot 50 \cdot 0.8 = 80\pi$

rel error $\quad \frac{dA}{A} \leq \frac{80\pi}{2500\pi} = .032 \quad (3.2\%)$