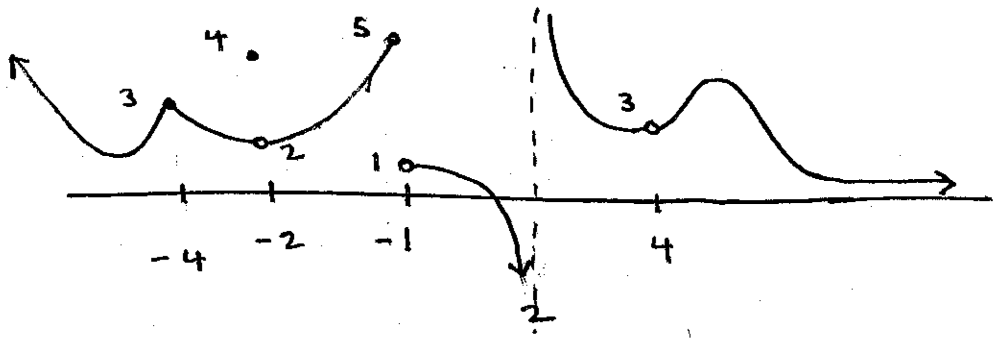
Exam 1 Practice Problems – Calculus I

Below is a set of practice problems covering each of the topics on the study guide. It is not enough to work only these problems. You should also review your quiz problems, pertinent homework problems, and end of chapter review in the text, but focus only on those problems mentioned in the study guide.

1. Identify the limits below given the graph of $f(x)$:



a) $\lim_{x\to-\infty} f(x) =$ ____ b) $\lim_{x\to-4} f(x) =$ ____ c) $\lim_{x\to-2} f(x) =$ ____ d) $\lim_{x\to-1} f(x) =$ ____

e) $\lim_{x\to2^-} f(x) =$ ____ f) $\lim_{x\to2^+} f(x) =$ ____ g) $\lim_{x\to2} f(x) =$ ____ h) $\lim_{x\to4} f(x) =$ ____

h) $\lim_{x\to\infty} f(x) =$ ____

2. Evaluate these limits

$$
\begin{aligned}
L_1 &= \lim_{x\to2} \frac{x^3-2x^2+x-2}{x^2+x-6} \\
L_2 &= \lim_{x\to2} \frac{x^2+3x-10}{x^2+x-6} \\
L_3 &= \lim_{x\to3} \frac{\sqrt{6+x}-3}{2x-6} \\
L_4 &= \lim_{x\to0} \frac{\sqrt{5+x}-\sqrt{5}}{x} \\
L_5 &= \lim_{x\to1} \frac{x^2-1}{|x-1|} \\
L_6 &= \lim_{x\to\infty} \frac{3x+2}{\sqrt{4x^2+1}} \\
L_7 &= \lim_{x\to-\infty} \frac{3x+2}{\sqrt{4x^2+1}} \\
L_8 &= \lim_{x\to\infty} \frac{\sqrt{36x^{10}+1}}{5x^5+3} \\
L_9 &= \lim_{x\to-\infty} \frac{\sqrt{36x^{10}+1}}{5x^5+3} \\
L_{10} &= \lim_{x\to-\infty} \frac{x^2}{\sqrt{x^4+17}} \\
L_{11} &= \lim_{x\to\infty} \frac{3x^2-7x+1}{5x^2+x+309} \\
L_{12} &= \lim_{x\to\infty} \frac{x^3+x^2+x+1}{x^2+2x+5} \\
L_{13} &= \lim_{x\to\infty} \frac{x^2+2x+5}{x^3+1712}
\end{aligned}
$$

3. (a) Trish the nutritionist is mixing a smoothie in a cylindrical bottle with a radius of exactly 10 cm. She wants the smoothie to have volume 3000 cm^3, but is willing to be off by ± 5 cm^3. (a) How high should be bottle be filled to get the desired volume? (b) How far from the ideal can the fluid height be and still have an acceptable volume? (c) Relate this to the definition of the limit, $L = \lim_{x \to a} f(x)$, by identifying x, $f(x)$, L, a, ϵ and δ.

   (b) Lionel the soccer player wants to inflate his soccer ball to have a volume of 523.6 in^3. (a) What is the ideal radius to attain this volume? (b) If the volume can be off by ± 10 in^3, how far from the ideal radius can the actual radius be and still lead to an acceptable volume? (c) Relate this to the definition of the limit, $L = \lim_{x \to a} f(x)$, by identifying x, $f(x)$, L, a, ϵ and δ.

4. Is $f(x)$ continuous at $x = 3$? Explain carefully why or why not.

   (a) $f(x) = \begin{cases} x, & \text{if } x \le 3; \\ x^2 - 6, & \text{if } x > 3; \end{cases}$

   (b) $f(x) = \begin{cases} x, & \text{if } x \le 3; \\ x^2 + 6, & \text{if } x > 3; \end{cases}$

5. (a) Let $f(x) = \begin{cases} x + 3, & \text{if } x < 0; \\ k, & \text{if } x = 0; \\ x - 4, & \text{if } x > 0; \end{cases}$

      i. Find the value of k that makes $f(x)$ continuous from the left at $x = 0$.
      ii. Find the value of k that makes $f(x)$ continuous from the right at $x = 0$.

   (b) Find the value of k that makes $f(x)$ continuous at $x = 1$ for $f(x) = \begin{cases} x^2 - k, & \text{if } \le 1; \\ 3x + 4k, & \text{if } x > 1; \end{cases}$

   (c) Find the values of a and b that make $f(x)$ continuous at $x = 0$ and $x = 2$ for $f(x) = \begin{cases} 5x + 2, & \text{if } x < 0; \\ x^2 + ax + b, & \text{if } 0 \le x < 2; \\ 3x + 1, & \text{if } x \ge 2; \end{cases}$

6. Sketch the graph of a function that satisfies the given properties (one sketch for (a) and one for (b)).

   (a) $\lim_{x \to 3^+} f(x) = 4$, $\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to -\infty} f(x) = -2$, $\lim_{x \to 5} f(x) = -\infty$, $\lim_{x \to \infty} f(x) = \infty$

   (b) $\lim_{x \to 1} f(x)$ DNE, $\lim_{x \to -2} f(x) = \infty$, $\lim_{x \to 4} f(x) = 6$

7. Use the definition of the derivative to find $f'(x)$ for $f(x)$.

   (a) $f(x) = 3x^2 + x + 2$

   (b) $f(x) = \sqrt{x}$

   (c) $f(x) = \dfrac{2}{x}$

   (d) $f(x) = \dfrac{1}{\sqrt{x}}$

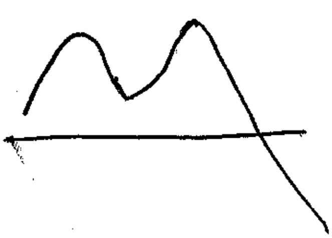
   (e) $f(x) = \dfrac{3}{x^2}$

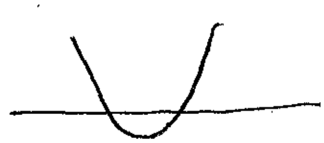
8. Find the equation of the tangent line ...

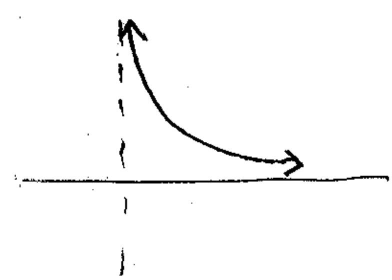
   (a) to $q(r)$ at $r = 2$ if $q'(r) = r^2 - 2r + 3$ and $q(2) = 7$.

   (b) to $w(t)$ at $t = -1$ if $w'(t) = \dfrac{3}{t + 4}$ and $w(-1) = 3$.

9. Sketch the graph of $f'(x)$ for each of the 3 graphs of $f(x)$







10. Use the Intermediate Value Theorem to to find an interval in which $f(x)$ has a root.

(a) $f(x) = x^2 - 3$

(b) $f(x) = x - e^x + 4$

(c) $f(x) = \frac{x^2 - 5x + 1}{x^2 + 1}$

11. BONUS LEARNING EXPERIENCE. Write the formula of a function that has vertical asymptotes at $x = 1$ and $x = 2$ and horizontal asymptote at $y = 3$.