Exam 1 Practice Problems

1 a) ∞, b) 3, c) 2, d) DNE, e) $-\infty$, f) $+\infty$

g) DNE, h) 3, i) 0

2 L_1 try $x=2$: $\frac{8-8+2-2}{4+2-6} = \frac{0}{0}$

so $f(x) = \frac{(x-2)[Q(x)]}{(x-2)(x+3)}$

Find $Q(x)$ by division

$$\begin{array}{r} x^2 \quad\quad + 1 \\ x-2 \;\big)\; x^3 - 2x^2 + x - 2 \\ -(x^3 - 2x^2) \quad\quad\quad \\ \hline 0 \quad + x - 2 \end{array}$$

so $L_1 = \lim_{x\to 2} \frac{x^2+1}{x+3} = \frac{5}{5} = 1$

$L_2 = \lim_{x\to 2} \frac{(x-2)(x+5)}{(x-2)(x+3)} = \lim_{x\to 2} \frac{x+5}{x+3} = \frac{7}{5}$

$L_3 = \lim_{x\to 3} \frac{\sqrt{6+x}-3}{2(x-3)} \cdot \frac{\sqrt{6+x}+3}{\sqrt{6+x}+3} = \lim_{x\to 3} \frac{(6+x)-9}{2(x-3)(\sqrt{6+x}+3)}$

$= \lim_{x\to 3} \frac{x-3}{2(x-3)(\sqrt{6+x}+3)} = \lim_{x\to 3} \frac{1}{2(\sqrt{6+x}+3)} = \frac{1}{2(3+3)} = \frac{1}{12}$

$L_4 = \lim_{x\to 0} \frac{\sqrt{5+x}-\sqrt{5}}{x} \cdot \frac{\sqrt{5+x}+\sqrt{5}}{x} = \lim_{x\to 0} \frac{(5+x)-(5)}{x} = \lim_{x\to 0} \frac{x}{x}$

$= \lim_{x\to 0} 1 = 1$ (you simplify before letting $x\to 1$)

L_5 : $\lim_{x\to 1^-} \frac{(x^2-1)}{-(x-1)} = \lim_{x\to 1^-} \frac{\cancel{(x-1)}(x+1)}{-\cancel{(x-1)}} = \frac{2}{-1} = -2$

$\lim_{x\to 1^+} \frac{\cancel{(x-1)}(x+1)}{+\cancel{(x-1)}} = \frac{2}{+1} = 2$ so L_5 DNE

$L_6 = \lim_{x\to\infty} \frac{3x+2}{\sqrt{4x^2+1}} \cdot \frac{1/x}{1/x} = \lim_{x\to\infty} \frac{3+2/x}{\sqrt{4x^2+1}\cdot 1/\sqrt{x^2}} = \lim_{x\to\infty} \frac{3+2/x}{\sqrt{4+1/x^2}}$

since $x \geq 0$

$= \frac{3+0}{\sqrt{4+0}} = \frac{3}{2}$

6

7e) $f(x) = \frac{3}{x^2}$

$f(x+h) = \frac{3}{(x+h)^2}$

$f(x+h) - f(x) = \frac{3}{(x+h)^2} - \frac{3}{x^2} = \frac{3x^2 - 3(x+h)^2}{(x+h)^2 \cdot x^2}$

$= \frac{3x^2 - [3x^2 + 6xh + h^2]}{(x+h)^2 x^2} = \frac{-6xh - h^2}{(x+h)^2 x^2}$

$\frac{f(x+h) - f(x)}{h} = \frac{-6x - h}{(x+h)^2 \cdot x^2}$

$f'(x) = \lim_{x \to 0} \frac{-6x - h}{(x+h)^2 x^2} = \frac{-6x}{x^2 \cdot x^2} = -\frac{6}{x^3}$ or $-6x^{-3}$

8) a) $Q'(2) = 4 - 4 + 3 = 3 = m$

$Q - Q_0 = m(r - r_0)$

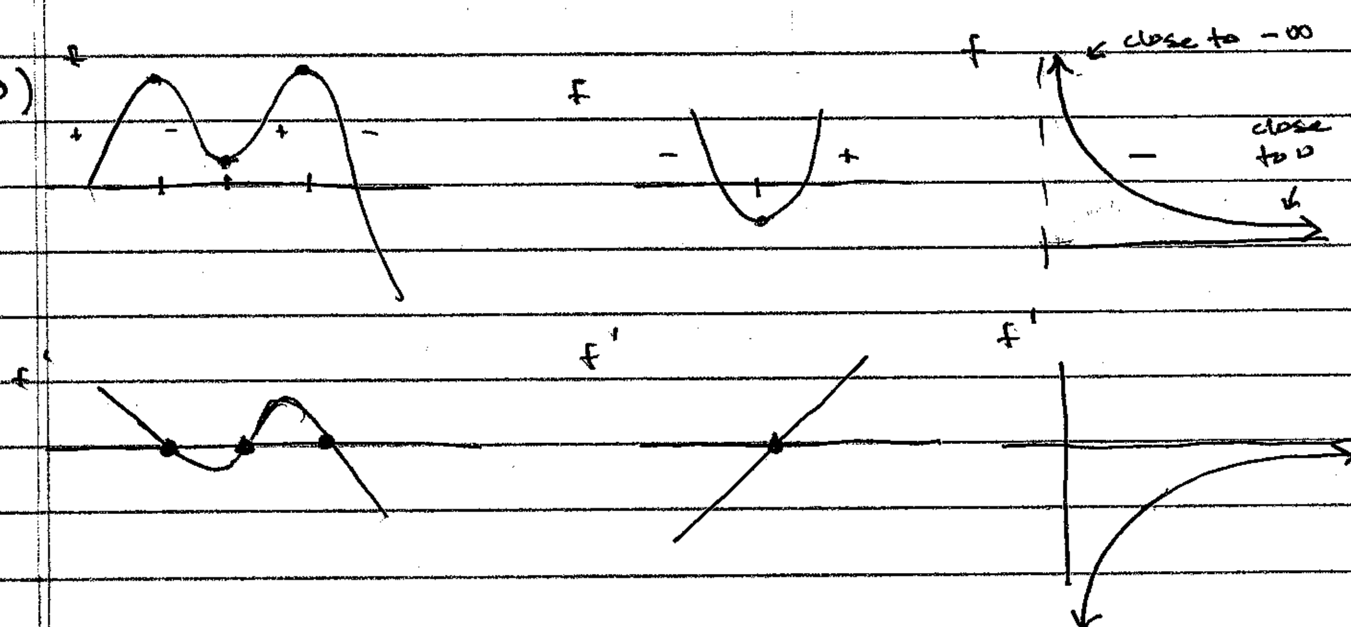
$Q - 7 = 3(r - 2)$ $\quad Q = 7 + 3(r-2)$

b) $w'(-1) = \frac{3}{-1+4} = -1$

$w - w_0 = m(t - t_0)$

$w - 3 = -1(t - (-1))$ $\quad w = 3 - (t+1)$

9)



5

7 a) $f(x) = 3x^2 + x + 2$

$f(x+h) = 3(x+h)^2 + (x+h) + 2 = 3x^2 + 6xh + 3h^2 + x + h + 2$

$f(x+h) - f(x) = 6xh + 3h^2 + h$

$\frac{f(x+h) - f(x)}{h} = 6x + 3h + 1$

$f'(x) = \lim_{h \to 0} 6x + 3h + 1 = 6x + 1$

b) $f(x) = \sqrt{x}$

$f(x+h) - f(x) = \sqrt{x+h} - \sqrt{x} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} = \frac{(x+h) - (x)}{\sqrt{x+h} + \sqrt{x}}$

$= \frac{h}{\sqrt{x+h} + \sqrt{x}}$

$\frac{f(x+h) - f(x)}{h} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$

$f'(x) = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$ or $\frac{1}{2} x^{-1/2}$

c) $f(x) = \frac{2}{x}$

$f(x+h) - f(x) = \frac{2}{x+h} - \frac{2}{x} = \frac{2}{x+h} \cdot \frac{x}{x} - \frac{2}{x} \cdot \frac{x+h}{x+h}$

$= \frac{2x - 2(x+h)}{x(x+h)} = \frac{-2h}{x(x+h)}$

$\frac{f(x+h) - f(x)}{h} = \frac{-2}{x(x+h)}$

$f'(x) = \lim_{h \to 0} \frac{-2}{x(x+h)} = -\frac{2}{x^2}$ or $-2x^{-2}$

d) $f(x) = \frac{1}{\sqrt{x}}$

$f(x+h) - f(x) = \frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}} = \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h} \cdot \sqrt{x}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$

$= \frac{(x) - (x+h)}{\sqrt{x+h} \cdot \sqrt{x}\,(\sqrt{x} + \sqrt{x+h})} = \frac{-h}{\sqrt{x+h} \cdot \sqrt{x} \cdot (\sqrt{x} + \sqrt{x+h})}$

$\frac{f(x+h) - f(x)}{h} = \frac{-1}{\text{stuff}}$

$f'(x) = \lim_{h \to 0} \frac{-1}{\sqrt{x+h}\sqrt{x}\,(\sqrt{x} + \sqrt{x+h})} = \frac{-1}{\underbrace{\sqrt{x}\sqrt{x}}_{x}\,(\sqrt{x} + \sqrt{x})} = \frac{-1}{x(2\sqrt{x})}$

$= -\frac{1}{2x^{3/2}}$

or $-\frac{1}{2} x^{-3/2}$

5b) $x^2 - k$ must match $3x + 4k$ at $x = 1$.

$1 - k = 3 + 4k \rightarrow -2 = 5k$, $k = -2/5$

5c) at $x = 0$, $5x + 2$ must match $x^2 + ax + b$

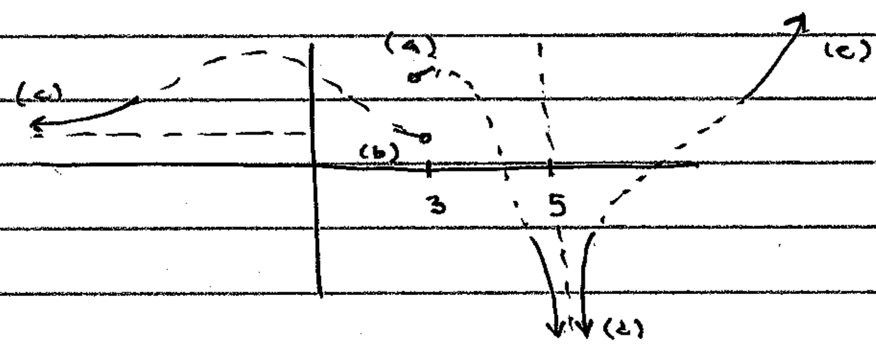
$0 + 2 = 0 + 0 + b \quad b = 2$

at $x = 2$, $x^2 + ax + \underline{b} = x^2 + ax + 2$ must match $3x + 1$

$4 + 2a + 2 = 6 + 1$

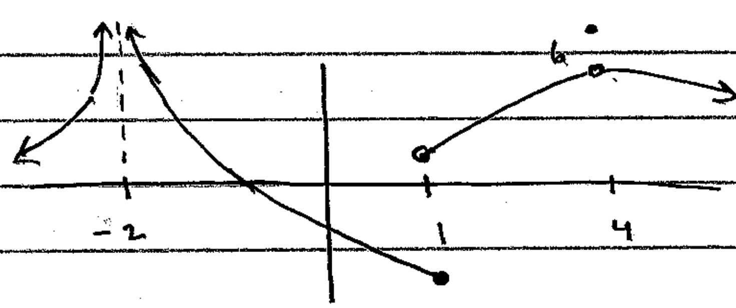
$2a = 1 \quad a = 1/2$

6) a)



required pieces: solid / fill-ins: dotted

b)

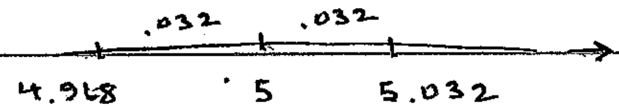


many options at $x = 1$
for limit DNE

3

3b $V = \frac{4}{3}\pi r^3$

- ideal radius $523.6 = \frac{4}{3}\pi r^3 \rightarrow r^3 = 125 \rightarrow r = 5.00$ in
- upper bound $533.6 = \frac{4}{3}\pi r^3 \rightarrow r^3 = 127.388 \rightarrow r = 5.032$ in
- lower bound $513.6 = \frac{4}{3}\pi r^3 \rightarrow r^3 = 122.613 \rightarrow r = 4.968$ in



radius can be off by .032 in

x = radius, $f(x)$ = volume, $L = 523.6\ in^3$, $a = 5.000$ in

$\varepsilon = 10\ in^3$ $\delta = .032$ in

4 a) from the left $f(x) = x \rightarrow 3$

from the right $f(x) = x^2 - 6 \Rightarrow 9 - 6 = 3$

(i) limit exists (ii) $f(3) = x\big|_{x=3} = 3$

(iii) limit equals function value $\rightarrow$ yes, f is continuous at $x=3$

b) from the left $f(x) = x \rightarrow 3$

from the right $f(x) = x^2 + 6 \rightarrow 9 + 6 = 15$

no, f is not continuous at $x=2$ because $\lim_{x\to 3} f(x)$ DNE

5 a) i) from the left, $x+3$ must match k at $x=0$

$0 + 3 = k$ $\quad k = 3$

ii) from the right, k must match $x-4$ at $x=0$:

$k = 0 - 4$ $\quad k = -4$

2

- $L_7 = \lim_{x\to-\infty} \frac{3x+2}{\sqrt{4x^2+1}} \cdot \frac{1/x}{-1/\sqrt{x^2}} = \lim_{x\to-\infty} \frac{3+2/x}{-\sqrt{4+1/x^2}} = -\frac{3}{2}$

  since $x<0$

- $L_8 = \lim_{x\to\infty} \frac{\sqrt{36x^{10}+1}}{5x^5+3} \cdot \frac{1/\sqrt{x^{10}}}{1/x^5} = \lim_{x\to\infty} \frac{\sqrt{36+1/x^{10}}}{5+3/x^5} = \frac{\sqrt{36}}{5} = \frac{6}{5}$

- L_9 only difference is $\frac{1}{x^5} = -\frac{1}{\sqrt{x^{10}}}$ since $x<0$, so $L_9 = -\frac{6}{5}$

- $L_{10} = \lim_{x\to-\infty} \frac{x^2}{\sqrt{x^4+17}} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to-\infty} \frac{x^2 \cdot 1/x^2}{\sqrt{x^4+17} \cdot 1/\sqrt{x^4}}$ $\quad x^2 > 0$ even if $x<0$

  $= \lim_{x\to-\infty} \frac{1}{\sqrt{1+17/x^4}} = \frac{1}{\sqrt{1+0}} = 1$

- $L_{11} = \lim_{x\to\infty} \frac{3x^2-7x+1}{5x^2+x+309} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{3-\frac{7}{x}+\frac{1}{x^2}}{5+\frac{1}{x}+\frac{309}{x^2}} = \frac{3}{5}$

- $L_{12} = \lim_{x\to\infty} \frac{x^3+x^2+x+1}{x^2+2x+5} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{x+1+\frac{1}{x}+\frac{1}{x^2}}{1+\frac{2}{x}+\frac{5}{x^2}} = \infty$

- $L_{13} = \lim_{x\to\infty} \frac{x^2+2x+5}{x^3+1712} \cdot \frac{1/x^3}{1/x^3} = \lim_{x\to\infty} \frac{1/x+2/x^2+5/x^3}{1+1712/x^3} = \frac{0+0+0}{1+0} = 0$

3 a) $V = \pi r^2 h \quad r=10$

- ideal height $3000 = \pi \cdot 100h \quad h = \frac{3000}{100\pi} = 9.549$ cm
- upper bound $3005 = \pi \cdot 100h \quad h = 9.565$ cm
- lower bound $2995 = \pi\, 100h \quad h = 9.533$ cm

.016 .016

9.533 9.549 9.565 → height can be off by .016 cm

x = height $\quad f(x)$ = volume

$L = 3000\ \text{cm}^3 \quad a = 9.549$ cm

$\varepsilon = 5\ \text{cm}^3 \quad \delta = .016$ cm

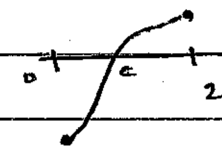
7

10 Find an x where $f(x) < 0$ and one where $f(x) > 0$

a) $f(x) = x^2 - 3$ • $x = 0$ $f(0) = -3$

• $x = 2$ $f(2) = 1$

there is a root in $(0,2)$



b) $f(x) = x - e^x + 4$ • $x = 0$ $f(0) = 0 - 1 + 4 = 3$

• $x = 5$ $f(5) = 5 - e^5 + 4$

$= 9 - e^5 \approx -139.4$

there is a root in $(0,5)$

c) $f(x) = \dfrac{x^2 - 5x + 1}{x^2 + 1}$ The denominator is always positive, so it doesn't play a role.

• $x = 0$ $f(0) = \dfrac{0 - 0 + 1}{0 + 1} = 1$

• $x = 1$ $f(1) = \dfrac{1 - 5 + 1}{1 + 1} = -\dfrac{3}{2}$

there is a root in $(0,1)$

11 To get the VAs, the denominator should be $(x-1)(x-2)$

To get the HA, the numerator should have the same net power of x: x^2

To get $y = 3$ put a 3 up top

$$y = \frac{3x^2}{(x-1)(x-2)}$$