THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Flash Cards
Trigonometric Equations
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Find all values of $t$ which solve the equation

$$\tan^2 t = 1.$$
Solve

\[ \sin (2t) = \sin t \]

where \( t \in [0, 2\pi] \).
Find all real numbers $x$ for which
\[ \sin^2 x = - \cos (2x). \]
Find the solution set for the equation
\[ \cot t = \cos t \]
on the interval \([0, 2\pi]\).
Find all values of $t$ for which
\[ \sin^2 t = 2 \sin t - 1. \]
Find all values of $\alpha \in [0^\circ, 360^\circ)$ for which

$$\cos(2\alpha) = 2\sin \alpha \cos \alpha.$$
Find all real solutions of the equation
\[ \cos (\pi + t) = \sin t. \]
Solve

\[ \sin \alpha \cos \alpha = 0 \]

where \( \alpha \in [0^\circ, 360^\circ) \).
Determine all $\alpha \in \left[0^\circ, 360^\circ \right)$ for which

$$\cos (2\alpha) = 2\sin^2 \alpha.$$
Solve

\[ 1 + \tan^2 \alpha = \cos \alpha \]

where \( \alpha \in [0^\circ, 360^\circ) \).
HINT

In order to find all values of $t$ which solve the equation

$$\tan^2 t = 1$$

recall that the period of the tangent function is $\pi$ so you may add $\pi k$ where $k$ is any integer to a solution to obtain another solution.
Answer:

\[
t = \frac{\pi}{4} + \pi k, \quad \frac{3\pi}{4} + \pi k \quad \text{for } k \in \mathbb{Z}
\]

Solution: To solve \(\tan^2 t = 1\) first take the square root of both sides. This gives

\[
\tan^2 t = 1
\]

\[\implies \tan t = \pm 1\]

\[\implies t = \frac{\pi}{4}, \frac{3\pi}{4} \quad t \in [0, \pi]\]

\[\implies t = \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k \quad \text{where } k \text{ is any integer.}\]
HINT

To solve

\[ \sin (2t) = \sin t \]

for \( t \) in the interval \([0, \pi]\) use the double angle identity

\[ \sin (2t) = 2 \sin t \cos t. \]
Answer: $t = 0, \frac{\pi}{3}, \text{ or } \pi$

Solution: To solve $\sin(2t) = \sin t$, subtract $\sin t$ from both sides of the equation then use the double angle identity for sine

$$\sin(2t) = 2\sin t \cos t.$$ 

This gives

$$\sin(2t) - \sin t = 0$$

$$\implies 2\sin t \cos t - \sin t = 0$$

$$\implies \sin t (2 \cos t - 1) = 0$$

$$\implies \sin t = 0, \cos t = \frac{1}{2}.$$ 

The first equation $\sin t = 0$ implies $t = 0, \pi$ for $t$ in the interval $[0, \pi]$ and the second equation $\cos t = \frac{1}{2}$ implies $t = \frac{\pi}{3}$ for $t$ in the interval $[0, \pi]$ Combining these gives the answer $t = 0, \frac{\pi}{3}$, or $\pi$. ▷
HINT

To determine all real numbers $x$ for which
\[ \sin^2 x = -\cos (2x) \]
use the half angle identity
\[ \sin^2 x = \frac{1 - \cos (2x)}{2}. \]
Answer: \[ x = \frac{\pi}{2} + \pi k \text{ for } k \in \mathbb{Z} \]

Solution: To solve \( \sin^2 x = -\cos (2x) \) first use the half angle identity

\[
\sin^2 x = \frac{1 - \cos (2x)}{2}.
\]

This gives

\[
\frac{1 - \cos (2x)}{2} = -\cos (2x)
\]

\[\implies 1 - \cos (2x) = -2 \cos (2x)\]

\[\implies 1 = -\cos (2x)\]

\[\implies 2x = \pi + 2\pi k \quad \text{where } k \text{ is any integer.}\]

This implies

\[ x = \frac{\pi}{2} + \pi k \text{ where } k \text{ is any integer.} \]
HINT

To find the solution set for the equation
\[ \cot t = \cos t \]
on the interval \([0, 2\pi]\) use the fact that
\[ \cot t = \frac{\cos t}{\sin t}. \]
Answer: \[ t = \frac{\pi}{2} + \pi k \text{ for any } k \in \mathbb{Z} \]

Solution: To solve \( \cot t = \cos t \) first use the identity \( \cot t = \frac{\cos t}{\sin t} \) to obtain

\[
\frac{\cos t}{\sin t} = \cos t
\]

\[
\Rightarrow \quad \frac{\cos t}{\sin t} - \cos t = 0
\]

\[
\Rightarrow \quad \cos t \left( \frac{1}{\sin t} - 1 \right) = 0
\]

\[
\Rightarrow \quad \cos t (\csc t - 1) = 0.
\]

\[
\Rightarrow \quad \cos t = 0 \text{ or } \csc t = 1.
\]

The first equation \( \cos t = 0 \) gives \( t = \frac{\pi}{2} + \pi k \) where \( k \) is any integer and the second equation \( \csc t = 1 \) gives \( t = \frac{\pi}{2} + 2\pi k \) where \( k \) is any integer, so the solution set is \( t = \left\{ \frac{\pi}{2} + \pi k \text{ where } k \text{ is any integer} \right\} \).
HINT

To find all values of $t$ for which

$$\sin^2 t = 2 \sin t - 1$$

factor the given expression.
Answer: \[ t = \frac{\pi}{2} + 2\pi k \text{ for any } k \in \mathbb{Z} \]

Solution: To solve
\[
\sin^2 t = 2 \sin t - 1
\]
first subtract \(2 \sin t - 1\) from both sides of the equation and factor to obtain
\[
\sin^2 t - 2 \sin t + 1 = 0
\]
\[\Rightarrow\] \[(\sin t - 1)^2 = 0\]
\[\Rightarrow\] \[\sin t - 1 = 0\]
\[\Rightarrow\] \[\sin t = 1.\]
This equation gives \(t = \frac{\pi}{2} + 2\pi k\) where \(k\) is any integer.
HINT

In order to determine the values of
\[ \alpha \in [0^\circ, 360^\circ) \]
for which
\[ \cos(2\alpha) = 2 \sin \alpha \cos \alpha \]
use the double angle formula for the sine function.
Answer: \[ \alpha = 22.5^\circ, 112.5^\circ \]

Solution: To solve

\[ \cos(2\alpha) = 2 \sin \alpha \cos \alpha \]

first make a substitution using the identity

\[ \sin (2\alpha) = 2 \sin \alpha \cos \alpha. \]

This gives

\[ \cos(2\alpha) = \sin (2\alpha) \]

\[ \implies \tan (2\alpha) = 1 \]

\[ \implies 2\alpha = 45^\circ, 225^\circ \]

\[ \implies \alpha = 22.5^\circ, 112.5^\circ \quad \text{for} \quad \alpha \in [0^\circ, 360^\circ) \]
HINT

In order to solve the equation
\[ \cos (\pi + t) = \sin t \]

remember
\[ \cos \pi = -1 \text{ and } \sin \pi = 0. \]
Answer: \[ t = -\frac{\pi}{4} + \pi k \text{ for } k \in \mathbb{Z} \]

Solution: To solve \( \cos(\pi + t) = \sin t \) first use the identity
\[
\cos(a + b) = \cos a \cos b - \sin a \sin b
\]
to obtain
\[
\cos \pi \cos t - \sin \pi \sin t = \sin t
\]
\[\implies -\cos t = \sin t \]
\[\implies -1 = \tan t \]
The last equation implies that \( t = -\frac{\pi}{4} + \pi k \) where \( k \) is any integer.
HINT

To solve

\[ \sin \alpha \cos \alpha = 0 \]

for all \( \alpha \in [0^\circ, 360^\circ) \) recall that two terms multiplied together to equal zero implies that at least one of the terms equals zero.
Answer: \( \alpha = 0^\circ, 90^\circ, 180^\circ, 270^\circ \)

Solution: To solve \( \sin \alpha \cot \alpha = 0 \) set each factor equal to zero and solve. This gives

\[
\sin \alpha = 0
\]

\[
\Rightarrow \quad \alpha = 0^\circ, 180^\circ \quad \text{or}
\]

\[
\cos \alpha = 0
\]

\[
\Rightarrow \quad \alpha = 90^\circ, 270^\circ.
\]

Therefore, when \( \alpha \in [0^\circ, 360^\circ) \) the solutions are

\[
\alpha = 0^\circ, 90^\circ, 180^\circ, 270^\circ.
\]
HINT

To solve

\[ \cos (2\alpha) = 2 \sin^2 \alpha \]

for \( \alpha \in [0^\circ, 360^\circ] \) use the double angle identity for the cosine function

\[ \cos (2\alpha) = 1 - 2 \sin^2 \alpha. \]
Answer: $\alpha = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

Solution: To solve $\cos (2\alpha) = 2 \sin^2 \alpha$ first use the double angle identity for the cosine function

$$\cos (2\alpha) = 1 - 2 \sin^2 \alpha.$$ 

This gives

$$1 - 2 \sin^2 \alpha = 2 \sin^2 \alpha \quad \Rightarrow \quad 1 - 4 \sin^2 \alpha = 0 \quad \Rightarrow \quad (1 - 2 \sin \alpha) (1 + 2 \sin \alpha) = 0 \quad \Rightarrow \quad \sin \alpha = \pm \frac{1}{2}.$$ 

Since $\alpha \in [0^\circ, 360^\circ)$, we have $\alpha = 30^\circ, 150^\circ, 210^\circ, 330^\circ.$
HINT

To solve

\[ 1 + \tan^2 \alpha = \cos \alpha \]

where \( \alpha \in [0^\circ, 360^\circ) \) use the identity

\[ 1 + \tan^2 \alpha = \sec^2 \alpha. \]
Answer: \[ \alpha = 180^\circ \]

Solution: To solve

\[ 1 + \tan^2 \alpha = \cos \alpha \]

first use the fundamental identity

\[ 1 + \tan^2 \alpha = \sec^2 \alpha. \]

This gives

\[
\sec^2 \alpha = \cos \alpha
\]

\[ \Rightarrow \]

\[
\sec^2 \alpha = \frac{1}{\sec \alpha}
\]

\[ \Rightarrow \]

\[
\sec^3 \alpha = 1
\]

\[ \Rightarrow \]

\[
\sec \alpha = 1.
\]

The \( \sec \alpha = 1 \) will be true whenever \( \cos \alpha = 1 \) since \( \sec \alpha = \frac{1}{\cos \alpha} \). For \( \alpha \in [0^\circ, 360^\circ) \) this implies that \( \alpha = 180^\circ \).