Memory Cards
Trigonometric identities
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and
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Express $\sin t$ in terms of a cofunction of the angle $t$ and name the type of identity.
Express $\cos t$ in terms of a cofunction of the angle $t$ and name the type of identity.
State two reciprocal or quotient identities for \( \tan t \).
True or False:

$$\cot t = \frac{\sin t}{\cos t}.$$ 

If you answer False provide a correct identity.
Express $\sec t$ in terms of another trigonometric function of the angle $t$ rad.
True or False:

\[ \csc t = \frac{1}{\sin t}. \]

If you answer False provide a correct identity.
Complete the following identities and name the type of identity:

\[
sin (-t) =
\]

\[
sec (-t) = ?
\]
True or False:

\[ \tan (-t) = -\tan t. \]

If you answer False provide a correct identity.
Complete the expression

\[ \cos (-t) = ? \]

and state type.
Express $\csc t$ in terms of the cosecant function.
True or False:

$$\cot (-t) = - \cot t.$$
Express the number 1 in terms of two trigonometric functions and state the type of identity.
Complete the expression
\[ \tan^2 t + 1 = ? \]
and state its type.
Express

$$\sec^2 t - 1$$

in terms of the tangent function.
Complete the expression

\[ 1 + \cot^2 t = ? \]

and state its type.
Express

\[ \csc^2 t - 1 \]

in terms of the cotangent function.
True or False: The ordered pair of real numbers \((\cos t, \sin t)\) represents a point on the unit circle for any radian measure \(t\).
True or False:

\[ \cos^2 t = (1 + \sin t) (1 - \sin t). \]
Complete the expression

$$\cos (\alpha + \beta) = ?$$

and state the identity.
Complete the expression
\[ \cos (\alpha + \beta) = ? \]
and state the identity.
Complete the expression

\[ \sin(\alpha + \beta) = ? \]

and state the identity.
Complete the expression

\[ \sin(\alpha - \beta) = ? \]

and state the identity.
Complete the expression

\[ \sin \alpha \cos \beta - \sin \beta \cos \alpha = ? \]

and state the identity.
Complete the expression

\[ \sin \alpha \cos \beta + \sin \beta \cos \alpha =? \]

and state the identity.
Complete the expression

$$\sin(\alpha + \beta) = ?$$

and state the identity.
Complete the expression
\[ \cos \alpha \cos \beta - \sin \alpha \sin \beta = ? \]
and state the identity.
Complete the expression

\[ \tan(\alpha + \beta) = ? \]

and state the identity.
Complete the expression

\[
\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = ?
\]

and state the identity.
Complete the expression

\[ \tan(\alpha - \beta) = ? \]

and state the identity.
Complete the expression

\[
\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = ?
\]

and state the identity.
Complete the expression
\[ \sin \left( \frac{\pi}{2} - t \right) = ? \]
and state the identity.
Complete the expression
\[ \cos \left( \frac{\pi}{2} - t \right) = ? \]
and state the identity.
Complete the expression
\[ \tan \left( \frac{\pi}{2} - t \right) = ? \]
and state the identity.
Complete the expression

\[ \cot \left( \frac{\pi}{2} - t \right) =? \]

and state the identity.
Complete the expression

$$\sec\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.
Complete the expression
\[ \csc \left( \frac{\pi}{2} - t \right) = ? \]
and state the identity.
Complete the expression
\[ \sin(2\alpha) = ? \]
and state the identity.
Complete the expression
\[ \cos(2\alpha) = ? \]
and state the identity.
Complete the expression

\[ \tan(2\alpha) = ? \]

and state the identity.
Complete the expression

\[
\sin \frac{\alpha}{2} = ?
\]

and state the identity.
Complete the expression

\[ \cos \frac{\alpha}{2} = ? \]

and state the identity.
Complete the expression
\[
\tan \left( \frac{\alpha}{2} \right)
\]
and state the identity.
Complete the expression
\[ \cos \alpha \cos \beta = ? \]
and state the identity.
Complete the expression

$$\sin \alpha \sin \beta = ?$$

and state the identity.
Complete the expression

$$\cos \alpha \sin \beta = ?$$

and state the identity.
True or False:

\[ \cos \alpha \sin \beta = \frac{1}{2} (\sin(\alpha + \beta) - \sin(\alpha - \beta)) \]
True or False:

\[ \sin \alpha \sin \beta = \frac{1}{2} (\sin(\alpha + \beta) - \sin(\alpha - \beta)) \]
True or False:

\[ \sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)) \]
True or False:

\[ \cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta)) \]
True or False:

\[
\cos \alpha \cos \beta = \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))
\]
HINT

Let $P(x, y)$ be the point on the unit circle centered at $(0, 0)$ that determines the standard position angle of measure $t$ rad. Then

$$\csc t = \frac{1}{y}$$
Answer: \[ \sin t = \frac{1}{\csc t} \]

This identity is a **reciprocal** and **quotient** identity.
HINT

Let \( P(x, y) \) be the point on the unit circle centered at \((0, 0)\) that determines the standard position angle of measure \( t \) rad. Then

\[
\sec t = \frac{1}{x}
\]
Answer: \[ \cos t = \frac{1}{\sec t} \]

This identity is a reciprocal and quotient identity.
HINT

The tangent function is related the sine and cosine function.
Answer: \[
\tan t = \frac{1}{\cot t} = \frac{\sin t}{\cos t}
\]
HINT

Recall that

\[ \tan t = \frac{\sin t}{\cos t} \]
Answer: False: \[ \cot t = \frac{\cos t}{\sin t} \]
HINT

The secant function is related to the cosine function.
Answer: \[ \sec t = \frac{1}{\cos t} \]

This identity is a **reciprocal** and **quotient** identity.
HINT

This question has two answers. Recall that \( \cot t = \frac{x}{y} \).
Answer: True
HINT

The sine function is odd. Recall that a function is odd if

\[ f(-x) = -f(x) \]

for all \( x \) in the domain of \( f \).
Answer:
\[
\sin(-t) = -\sin(t) \quad \text{and} \quad \sec(-t) = \sec t
\]

These are symmetric identities.
HINT

The graph of the tangent function is given below.
Answer: \[ True \]

This is a symmetric identity and it suggests that the tangent function is odd.
HINT

Recall that the cosine function is even. A function $f$ is even if

$$f(-x) = f(x)$$

for all $x$ in the domain of $f$. 
Answer: $True$

This is a symmetric identity.
HINT

The graph of the cosecant function is given below.
Answer: $\csc (-t)$

This is a symmetric identity.
HINT

Recall the symmetric identities.
Answer: True
HINT

Let \((x, y)\) be the point on the unit circle with center \((0, 0)\) that determines the standard position angle \(t\) rad. Then

\[x^2 + y^2 = 1\]
Answer: \( \sin^2 t + \cos^2 t = 1 \)

This is the first of the Pythagorean identities.
HINT

Dividing through the identity

$$\sin^2 t + \cos^2 t = 1$$

by $\cos^2 t$ yields

$$\frac{\sin^2 t}{\cos^2 t} + \frac{\cos^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}.$$
Answer: \[ \tan^2 t + 1 = \sec^2 t \]

This is a Pythagorean identity.
HINT

Use a Pythagorean identity.
Answer: $\sec^2 t - 1 = \tan^2 t$
HINT

Dividing through the identity
\[ \sin^2 t + \cos^2 t = 1 \]
by \( \sin^2 t \) yields
\[
\frac{\sin^2 t}{\sin^2 t} + \frac{\cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}.
\]
Answer: \[ 1 + \cot^2 t = \csc^2 t \]

This is a Pythagorean identity.
HINT

Use a Pythagorean identity.
Answer: $\csc^2 t - 1 = \cot^2 t$
HINT

Recall that the equation of the unit circle is

\[ x^2 + y^2 = 1. \]
The Pythagorean identity

\[ \sin^2 t + \cos^2 t = 1 \]

verifies that \((\cos t, \sin t)\) lies on the unit circle since \(x = \cos t\) and \(y = \sin t\) satisfies

\[ x^2 + y^2 = 1. \]
HINT

Recall the Pythagorean identities.
Answer: True

\[ \cos^2 t = 1 - \sin^2 t = (1 + \sin t)(1 - \sin t) \]
HINT

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - ? \]
Answer:

$$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

This is the sum formula for the cosine function.
HINT

\[ \cos (\alpha - \beta) = \cos \alpha \cos \beta + ? \]
Answer:

\[
\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta
\]

This is the difference formula for the cosine function.
HINT

\[ \sin(\alpha + \beta) = ? + \sin \beta \cos \alpha \]
Answer:

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha \]

This is the sum formula for the sine function.
HINT

\[ \sin(\alpha - \beta) =? - \sin \beta \cos \alpha \]
Answer:
\[ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha \]
This is the difference formula for the sine function.
HINT

Recall the difference formula for the sine function.
Answer:
\[
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha
\]

This is the difference formula for the sine function.
HINT

Recall the sum formula for the sine function.
Answer:

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha \]

This is the difference formula for the sine function.
HINT

\[ \sin(\alpha + \beta) = ? + \sin \beta \cos \alpha \]
Answer:
\[\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha\]

This is the sum formula for the sine function.
HINT

Recall the sum formula for the cosine function.
Answer:

\[
\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]

This is the sum formula for the cosine function.
HINT

\[ \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{?} \]
Answer: \[ \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \]

This is the sum formula for the tangent function.
HINT

Recall the sum formula for the tangent function.
Answer: \[ \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \]

This is the sum formula for the tangent function.
HINT

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{?}$$
Answer: \[ \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \]

This is the difference formula for the tangent function.
HINT

Recall the difference formula for the tangent function.
Answer: \[ \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \]

This is the sum formula for the tangent function.
HINT

If you have forgotten this you may construct the answer by applying the difference formula for the sine function.
Answer: \[ \sin \left( \frac{\pi}{2} - t \right) = \cos t \]

The is a cofunction identity.
HINT

If you have forgotten this you may construct the answer by applying the difference formula for the cosine function.
Answer: $\cos \left( \frac{\pi}{2} - t \right) = \sin t$

The is a cofunction identity.
HINT

Observe that

\[ \tan \left( \frac{\pi}{2} - t \right) = \frac{\sin \left( \frac{\pi}{2} - t \right)}{\cos \left( \frac{\pi}{2} - t \right)} . \]
Answer: \[
\tan \left( \frac{\pi}{2} - t \right) = \cot t
\]

The is a cofunction identity.
HINT

Observe that

$$\cot \left( \frac{\pi}{2} - t \right) = \frac{\cos \left( \frac{\pi}{2} - t \right)}{\sin \left( \frac{\pi}{2} - t \right)}.$$
Answer: \( \cot \left( \frac{\pi}{2} - t \right) = \tan t \)

The is a cofunction identity.
HINT

Observe that

$$\sec \left( \frac{\pi}{2} - t \right) = \frac{1}{\cos \left( \frac{\pi}{2} - t \right)}.$$
Answer: \[ \sec \left( \frac{\pi}{2} - t \right) = \csc t \]

The is a cofunction identity.
HINT

Observe that

\[
\csc \left( \frac{\pi}{2} - t \right) = \frac{1}{\sin \left( \frac{\pi}{2} - t \right)}.
\]
Answer: \[ \csc \left( \frac{\pi}{2} - t \right) = \sec t \]

The is a cofunction identity.
HINT

If you have forgotten consider

\[ \sin (\alpha + \alpha) \]
Answer: \[ \sin(2\alpha) = 2 \sin \alpha \cos \alpha \]

This is a double angle formula.
HINT

This question has three answers.
Answer:

\[
\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha
= 1 - 2 \sin^2 \alpha
= 2 \cos^2 \alpha - 1
\]

These are the three double angle formulas for the cosine function.
HINT

If you have forgotten consider

\[ \tan (\alpha + \alpha) \]
Answer: \[ \tan(2\alpha) = \frac{2\tan \alpha}{1-\tan^2 \alpha} \]

This is a double angle formula.
HINT

If necessary you may use the double angle formula

$$\cos \alpha = \cos \left(2 \frac{\alpha}{2}\right) = 1 - \sin^2 \frac{\alpha}{2}$$
Answer: \[ \sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} \]

This is a half angle formula.
HINT

If necessary you may use the double angle formula

\[ \cos \alpha = \cos \left( \frac{2\alpha}{2} \right) = 2 \cos^2 \frac{\alpha}{2} - 1 \]
Answer: \[ \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}} \]

This is a half angle formula.
HINT

This question has three answers.
Answer:

\[ \tan \left( \frac{\alpha}{2} \right) = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \]

\[ = \frac{\sin \alpha}{1 + \cos \alpha} \]

\[ = \frac{1 - \cos \alpha}{\sin \alpha} \]

This are the half angle formulas for the tangent function.
HINT

Consider the sum

\[ \cos(\alpha + \beta) + \cos(\alpha - \beta) \]
Answer:

\[ \cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta)) \]

This is a product formula.
HINT

Consider the sum

\[ \cos(\alpha - \beta) - \cos(\alpha + \beta) \]
Answer:

$$\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

This is a product formula.
HINT

Consider the sum

\[ \sin(\alpha + \beta) - \sin(\alpha - \beta) \]
Answer:

\[
\cos \alpha \sin \beta = \frac{1}{2} \left( \sin(\alpha + \beta) - \sin(\alpha - \beta) \right)
\]

This is a product formula.
HINT

If you are not sure use the sum formulas on the expression

\[ \cos \alpha \sin \beta = \frac{1}{2} (\sin(\alpha + \beta) - \sin(\alpha - \beta)) . \]
Answer: \( True \)

This is a product formula.
HINT

If you are not sure use the sum formulas on the

\[ \frac{1}{2} \left( \sin(\alpha + \beta) - \sin(\alpha - \beta) \right). \]
Answer: \textit{False}
HINT

If you are not sure use the sum formulas on the expression

\( \frac{1}{2} \left( \cos(\alpha - \beta) - \cos(\alpha + \beta) \right) \).
Answer: True

This is a product formula. \( \cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta)) \)
HINT

If you are not sure use the sum formulas on the expression

$$\frac{1}{2} (\cos(\alpha + \beta) + \cos(\alpha - \beta)) .$$
Answer: True

This is a product formula.
HINT

If you are not sure use the sum formulas on the expression

$$\frac{1}{2} \left( \sin(\alpha + \beta) + \sin(\alpha - \beta) \right).$$
Answer: \textit{False}