Review Questions: Also study quizzes and homework.

I. Let \( \mathbf{a} = \langle \frac{1}{2}, -1, 0 \rangle \), \( \mathbf{b} = \langle 4, 1, -1 \rangle \), and
\[
\mathbf{r}(t) = \langle e^{2t}, \ln(t + 1), t + \sec(t) + 2 \rangle.
\]

1. Find \( \text{comp}_b \mathbf{a} \).

2. Find \( \cos \theta \) where \( \theta \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \).

3. Find the tangent vector to \( \mathbf{r}(t) \) at \( t = 0 \).

4. Find \( (\mathbf{b} \times \mathbf{a}) \cdot \mathbf{a} \).

5. Find the unit tangent to \( \mathbf{r}(t) \) at \( t = 0 \).

6. Find \( \frac{1}{2} \mathbf{b} - 2 \mathbf{a} \).

7. Find a vector parallel to \( \mathbf{a} \) but twice as long as \( \mathbf{a} \).

8. Find the area of the parallelogram with sides the vectors \( \mathbf{a} \) and \( \mathbf{b} \).

II. Let \( P = (0, -1, 2) \), \( Q = (2, 1, -1) \), and
\[
\mathbf{r}(t) = \langle \cos(1 - e^t), t \ln(1 - t), t^2 + 2t \rangle.
\]

1. Find symmetric equations for the line through points \( P \) and \( Q \).
   
   The vector \( \overrightarrow{PQ} = \) ________________.

2. Find parametric equations for the tangent line to \( \mathbf{r}(t) \) at \( t = 0 \).
   
   The vector \( \mathbf{r}'(0) = \) ________________.
3. Find parametric equations for the line through $P$ and perpendicular to the plane $7 - 3z = 0$.
   The normal vector of the plane $7 - 3z = 0$ is: ________________.

4. Find the plane containing $P$ and perpendicular to $\vec{QP}$.
   The normal vector $\vec{QP} =$ ________________.

5. Find the plane through the point $Q$ and perpendicular to $\vec{QP}$.
   The normal vector $\vec{r}(0) =$ ________________.

   For the following 3:
   Let $\vec{r}(t) = \langle e^{2t}, 2\tan t, \ln(t+1) \rangle$.

6. Find the normal component of acceleration, $a_N(0)$ of $\vec{r}(t)$.
   $\vec{r}''(0) = $ ________________.

7. Find the curvature $\kappa(0)$ of $\vec{r}(t)$.
   $\vec{r}'(0) \times \vec{r}''(0) = $ ________________.

8. Find the tangential component of acceleration, $a_T(0)$ of $\vec{r}(t)$.
   $\vec{r}'(0) = $ ________________.

9. Given $\vec{r}(t) = \langle 5e^{2\tan t}, 1 + \frac{t^3}{t+1}, t^3 \rangle$.
   Find the tangent line to the curve $\vec{r}(t)$ at $t = 0$. Give parametric equations for the line.

10. Given $P = (1, 2, 2)$; $Q = (0, 1, 0)$; $R = (0, 2, 2)$.
    Find the plane through these three points. Simplify the plane equation so that all constants are combined on the right hand side.

11. Given $\vec{a} = \langle 1, 1, 3 \rangle$ and $\vec{b} = \langle 1, 0, 0 \rangle$.
    Find the area of the triangle with these vectors (arrows) as two of its sides. Give your answer as a real number; you may leave any roots as you found them.
12. Given

\[ \mathbf{r}(t) = \langle \ln t, 2, t^2 + t \rangle \] and \[ \mathbf{r}'(t) = \langle \frac{1}{t}, 0, 2t + 1 \rangle \] and \[ \mathbf{r}''(t) = \left\langle \frac{-1}{t^2}, 0, 2 \right\rangle \]

Find \( v(1), a(1), a_T(1), a_N(1), \kappa(1) \).

13. Given

\[ \mathbf{r}'(2) = \langle 0, 0, 3 \rangle, \mathbf{T}'(2) = \langle 1, 3, 0 \rangle, \] and \( a_T(2) = 5 \)

Find \( N(2), a(2), a_N(2), \kappa(2) \).

14. Given

\[ \mathbf{r}'(2) = \langle 6, 0, 3 \rangle, \mathbf{r}''(2) = \langle 1, 1/3, 0 \rangle \).

Find \( T(2), a(2), a_N(2), a_T(2), \kappa(2), N(2) \).

15. Find the x-value of the maximum curvature of \( y = x^6 \).

16. Find the x-value of the maximum curvature of \( y = 3e^x \).

17. Study all quiz questions!