1. Write \( \frac{5 - 3i}{(2 + i)(1 - i)} \) in the form \( a + ib \).  

2. Use the definition of \( e^z \) to prove that \( |e^z| = e^x \).  

3. Use polar coordinates to find the exact value of \( (1 - i\sqrt{3})^{30} \) in the form \( a + ib \).  

4. Write the 4th roots of \(-16\) in the form \( a + ib \).
5. *Explain* the difference between:

"\( f(z) \) is *differentiable* at \( z = z_0 \)" and "\( f(z) \) is *analytic* at \( z = z_0 \)"

6. Find \( f'(z) \), if \( f(z) = \frac{\exp(z^3 + 5z)}{z^4 + 1} \).

7. Use Euler’s formula to derive the trigonometric expansions of \( \cos(\theta + \phi) \) and \( \sin(\theta + \phi) \).

8. Use the limit definition to show that \( f(z) = z \) is not differentiable at \( z = 0 \).
9. Let $v = xy + e^{-x} \cos y$.
   a) Show that $v$ is harmonic.

b) Find a function $u$ such that $u + iv$ is analytic.

10. Consider the function $f(z) = \frac{z - i}{z + i}$.
   a) Prove that, if $z$ is real, $|f(z)| = 1$.

b) By the above, $f(z)$ maps the real axis onto the unit circle $|z| = 1$. What does this function do to the upper-half plane $Im \ z \geq 0$?