1. Evaluate the following limit, if it exists: \( \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \), for \( x \neq 0 \).

\[ \text{Solution:} \quad \lim_{h \to 0} \frac{1}{(x+h)^2} - \frac{1}{x^2} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2} = \lim_{h \to 0} \frac{-h(2x+h)}{h(x+h)^2 x^2} = \lim_{h \to 0} \frac{-2x}{(x+h)^2 x^2} = -\frac{2x}{x^2} \]

2. Prove that \( \lim_{x \to 0} x^4 \cos \left( \frac{2}{x} \right) = 0 \).

\[ \text{Solution:} \quad \text{We have that} \lim_{x \to 0} x^4 = 0 \text{ and } -1 \leq \cos \left( \frac{2}{x} \right) \leq 1, \text{ so the limit is 0 by the Squeeze Theorem.} \]

3. Use the \( \epsilon - \delta \) definition of the limit to prove that \( \lim_{x \to -2} \left( \frac{x}{4} + 3 \right) = \frac{5}{2} \).

\[ \text{Solution:} \quad \text{Let } \epsilon > 0 \text{ be given. Choose } \delta = 4\epsilon. \text{ Then,} \]
\[ |x - (-2)| < \delta \]
\[ \Rightarrow |x + 2| < 4\epsilon \]
\[ \Rightarrow \left| \frac{x+2}{4} \right| < \epsilon \]
\[ \Rightarrow \left| \left( \frac{x}{4} + 3 \right) - \frac{5}{2} \right| < \epsilon \]

4. Use the \( \epsilon - \delta \) definition of the limit to prove that \( \lim_{x \to 2} (x^2 - 4x + 5) = 1 \).

\[ \text{Solution:} \quad \text{Let } \epsilon > 0 \text{ be given. Choose } \delta = \sqrt{\epsilon}. \text{ Then,} \]
\[ |x - 2| < \delta \]
\[ \Rightarrow |x - 2| < \sqrt{\epsilon} \]
\[ \Rightarrow |x - 2|^2 < \epsilon \]
\[ \Rightarrow |(x - 2)^2| < \epsilon \]
\[ \Rightarrow |x^2 - 4x + 4| < \epsilon \]
\[ \Rightarrow |(x^2 - 4x + 5) - 1| < \epsilon \]

5. Find the numbers at which
\[ f(x) = \begin{cases} 
1 + x^2 & \text{if } x \leq 0 \\
2 - x & \text{if } 0 < x \leq 2 \\
x - 2 & \text{if } 2 < x 
\end{cases} \]
is discontinuous. Determine, at these points, whether \( f(x) \) is continuous from the left, from the right, or neither.

**Solution:** Clearly, since \( f(x) \) is a polynomial in each interval \((-\infty, 0)\), \((0, 2)\) and \((2, \infty)\), it is continuous there.

At \( x = 0 \), \( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (1 + x^2) = 1 \), \( f(0) = 1 + x^2 \big|_{x=0} = 1 \), and \( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2 - x) = 2 \).

Thus, \( f(x) \) is continuous from the left at \( x = 0 \), but not continuous there.

At \( x = 2 \), \( \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2 - x) = 0 \), \( f(2) = 2 - x \big|_{x=2} = 0 \), and \( \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x - 2) = 0 \).

Thus, \( f(x) \) is continuous at \( x = 2 \).