1. Use a Riemann sum with \( n = 6 \) sub-intervals to approximate \( \int_{1}^{4} e^{-x^2} \, dx \), with
   (a) Left endpoints
   (b) Right endpoints

   Solution: We have \( \Delta x = \frac{4 - 1}{6} = 0.5 \), from which \( x_0 = 1.0, \ x_1 = 1.5, \ x_2 = 2.0, \ x_3 = 2.5, \ x_4 = 3.0, \ x_5 = 3.5, \ x_6 = 4.0 \). With \( f(x) = e^{-x^2} \), the corresponding function values (rounded to 6 decimal places), are \( f(x_0) = 0.367879, \ f(x_1) = 0.105399, \ f(x_2) = 0.018316, \ f(x_3) = 0.001930, \ f(x_4) = 0.000123, \ f(x_5) = 0.000005 \) and \( f(x_6) = 0.0 \)

   (a) Using left endpoints, the sum is
   \[ \sum_{i=1}^{6} f(x_{i-1}) \, \Delta x = 0.246826 \]

   (b) Using right endpoints, the sum is
   \[ \sum_{i=1}^{6} f(x_i) \, \Delta x = 0.062887 \]

2. Write \( \lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{1 + 2 \left(2 + \frac{4i}{n}\right)} \cdot \frac{4}{n} \) as an integral.

   Solution: We have \( \Delta x = \frac{4}{n} \), so the interval of integration has length 4.

   We also have \( f(x_i^*) = \sqrt{1 + 2 \left(2 + \frac{4i}{n}\right)} \), so the obvious choice is \( x_i^* = x_i = 2 + \frac{4i}{n} \), and
   \( f(x) = \sqrt{1 + 2x} \), from which \( x_0 = 2 \) and \( x_n = 6 \).

   That is, the limit can be written as
   \[ \int_{2}^{6} \sqrt{1 + 2x} \, dx. \]

3. Define \( f(x) = \begin{cases} 3 & \text{if } x \leq 2 \\ 5 - x & \text{if } 2 < x \end{cases} \)

   (a) Sketch the graph of \( f(x) \) for \( 0 \leq x \leq 6 \).

   (b) Evaluate \( \int_{0}^{6} f(x) \, dx \) by expressing it in terms of areas.
Solution:

(a) The graph is:

(b) The integral is the difference of the trapezoidal area above the $x$-axis, which has size $\frac{5 + 2}{2} \cdot 3 = \frac{21}{2}$, and the triangular area below, of area $\frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$.

So, $\int_0^6 f(x) \, dx = \frac{21}{2} - \frac{1}{2} = 10$.

4. Define $g(x) = \int_{\ln x}^{e^x} \sin(t^2) \, dt$. Use the Fundamental Theorem of Calculus to find $g'(x)$.

Solution: Let $G(x) = \int_0^x \sin(t^2) \, dt$, so that $G'(x) = \sin(x^2)$.

Now, $g(x) = G(e^x) - G(\ln x)$. So, by the Chain Rule,

$$g'(x) = G'(e^x) \cdot e^x - G'(\ln x) \cdot \frac{1}{x} = e^x \sin(e^x) - \frac{\sin((\ln x)^2)}{x}.$$

5. A particle moves on the $x$-axis with velocity $v(t) = t^2 - 4$ m/s for $t \geq 0$.

(a) Find the net distance travelled for $1 \leq t \leq 5$ secs.

(b) Find the total distance travelled for $1 \leq t \leq 5$ secs.

Solution:

(a) The net distance travelled is

$$\int_1^5 v(t) \, dt = \int_1^5 (t^2 - 4) \, dt = \frac{t^3}{3} - 4t \bigg|_1^5 = \left( \frac{5^3}{3} - 4(5) \right) - \left( \frac{1^3}{3} - 4(1) \right) = \frac{76}{3}.$$

(b) The total distance travelled is $\int_1^5 |v(t)| \, dt = \int_1^2 (4 - t^2) \, dt + \int_2^5 (t^2 - 4) \, dt$

$$= 4t - \frac{t^3}{3} \bigg|_1^2 + \frac{t^3}{3} - 4t \bigg|_2^5 = \left( \frac{16}{3} - \frac{11}{3} \right) + \left( \frac{65}{3} - \left( \frac{-16}{3} \right) \right) = \frac{86}{3}.$$
6. Use the substitution $u = 3 - x$ to evaluate $\int x^2\sqrt{3-x} \, dx$.

Solution: We have $u = 3 - x$, so $x = 3 - u$ and $dx = -du$.

Hence, $\int x^2\sqrt{3-x} \, dx = \int (3-u)^2 u^{1/2} (-du) = \int (-9 + 6u - u^2) u^{1/2} du$

$= \int (-9u^{1/2} + 6u^{3/2} - u^{5/2}) du = -\frac{9u^{3/2}}{3/2} + \frac{6u^{5/2}}{5/2} - u^{7/2}/7/2 + C$

$= -6(3-x)^{3/2} + \frac{12}{5}(3-x)^{5/2} - \frac{2}{7}(3-x)^{7/2} + C$