1. State:
   
   (a) The Cauchy Integral Formula for Derivatives

   (b) The Maximum Modulus Principle

2. Let $C$ be the contour which consists of the line segment from $z = 0$ to $z = 1$, and from $z = 1$ to $z = 1 + i$. Evaluate $\int_C \overline{z} \, dz$.  

   10 points
3. Let $C$ denote the positively oriented circle $|z| = 1$, traversed once.

Define $g(w) = \int_C \frac{\sin(z^3)}{(z-w)^2} \, dz$

(a) Evaluate $g(w)$ for $|w| > 1$, and justify your answer.

(b) Evaluate $g(w)$ for $|w| < 1$, and justify your answer.

4. Consider the function $f(z) = \frac{z^2 - z}{3 - z}$.

Find the Laurent series for $f(z)$ valid in :

(a) $0 < |z| < 3$

(b) $3 < |z| < \infty$
5. (a) Show that \( f(z) = \frac{\cos \left( \frac{1}{z^2} \right)}{z} \) has an essential singularity at \( z = 0 \).

(b) Determine what kind of singularity \( g(z) = \frac{\sin z}{z^{14}} \) has at \( z = 0 \), and find the residue at this point.

10 points

10 points
6. Let $C$ denote the positively-oriented circle $|z - i| = \frac{3}{2}$, traversed once, and let $f(z) = \frac{z^2}{z^4 - 1}$

(a) Find all isolated singularities of $f(z)$ which lie inside $C$.

(b) Find the residues of $f(z)$ at these singularities.

(c) Evaluate $\int_C f(z) \, dz$