Calculus I : Fall 2013  Final Exam: 12/9/13  Dr. Tim Norfolk

INSTRUCTIONS : Show all of your work, and give exact answers.

1. Let \( f(x) = \begin{cases} 
2x - 2 & \text{if } 0 \leq x \leq 3 \\
10 - 2x & \text{if } 3 < x \leq 6 
\end{cases} \)

(a) Sketch the graph of \( f(x) \).

(b) Evaluate \( \int_{0}^{6} f(x) \, dx \)

(c) Evaluate \( \int_{0}^{6} \left| f(x) \right| \, dx \)

2. Evaluate \( \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{1 + \frac{6k}{n}} \cdot \frac{6}{n} \), by writing it as an integral.
3. A particle moves on the $x$-axis with velocity $v(t) = \cos(t) \text{ m/s}.$

   (a) Find the net distance travelled for $0 \leq t \leq \pi.$

   (b) Find the total distance travelled for $0 \leq t \leq \pi.$

4. Define $g(x) = \int_{x}^{x^3} e^{t^2} \, dt.$ Find $g'(x).$

5. Evaluate the following integrals:

   (a) $\int_{1}^{2} \left(3x - \frac{1}{\sqrt{x}}\right)^2 \, dx$
(b) \( \int_1^3 x e^{3x^2} \, dx \)

(c) \( \int x \sqrt{1 + x} \, dx \)

(d) \( \int \frac{2x}{x + 1} \, dx \)

6. Evaluate \( \lim_{h \to 0} \frac{4e^{8+2h} - 4e^8}{h} \), by writing it as a derivative. You may NOT use l’Hopital’s Rule.
7. Sketch the graph of a function $f(x)$ which satisfies:

(a) $f(-2) = 1; f(0) = 2; f(3) = 0$
(b) $\lim_{x \to -\infty} f(x) = 0; \lim_{x \to 2^-} f(x) = -\infty; \lim_{x \to 2^+} f(x) = +\infty$
(c) $f'(0) = 0; f'(3) = 0; f'(x) > 0$ on $(-\infty, 0) \cup (3, \infty); f'(x) < 0$ on $(0, 2) \cup (2, 3)$
(d) $f''(-2) = 0; f''(x) > 0$ on $(-\infty, -2) \cup (2, \infty); f''(x) < 0$ on $(-2, 2)$

8. Let $f(x) = \begin{cases} x^2 - ax & \text{if } x \leq a \\ 3x + 1 & \text{if } a < x \end{cases}$

Find the value(s) of $a$ for which $f(x)$ is continuous everywhere. Show the necessary one-sided limits.
9. Let \( y = \frac{\tan^{-1} x}{x^2} \)

(a) Find \( y' \)

(b) Find the equation of the tangent line at the point \( x = 1 \) Use exact values.

10. We wish to construct a rectangular pen of total area 360 \( m^2 \). Because of zoning restrictions, the one side of the pen along a road requires fencing which costs $4 per meter, while the other 3 sides only cost $1 per meter.

Find the dimensions of the pen which minimize the cost.