1. Suppose that $g''(x) < 0$ on $[1,4]$, and $g'(3) = 0$.

For each of the following, only circle ‘TRUE’ if the statement MUST be true.

(a) TRUE FALSE $g(x)$ is continuous on $[1,4]$.
(b) TRUE FALSE $g(2) = g(4)$.
(c) TRUE FALSE $x = 3$ is a local minimum of $g(x)$.
(d) TRUE FALSE $x = 3$ is a local maximum of $g(x)$.
(e) TRUE FALSE $x = 3$ is an absolute maximum of $g(x)$ on $[1,4]$.

2. Let $f(x) = x(1 - x^3)^{1/3}$.

(a) Show that $f'(x) = (1 - x^3)^{-2/3}(1 - 2x^3)$.

(b) Find any critical points of $f(x)$, and classify them.
3. Given the following, sketch the graph of $f(x)$:

(a) $f(-3) = 1; f(-2) = 0; f(0) = 0$
(b) $\lim_{x \to -\infty} f(x) = 0; \lim_{x \to -1^-} f(x) = +\infty; \lim_{x \to -1^+} f(x) = -\infty$
(c) $f'(x) > 0$ on $(-\infty, -3) \cup (-2, -1) \cup (-1, 0) \cup (0, +\infty); f'(x) < 0$ on $(-3, -2); f'(-3)$ is undefined; $f'(0) = 0$
(d) $f''(x) > 0$ on $(-\infty, -3) \cup (-3, -1) \cup (0, +\infty), f''(x) < 0$ on $(-1, 0)$

4. Evaluate the following limits:

(a) $\lim_{x \to 1} \frac{x \ln(x) - x + 1}{(x - 1)^2}$

(b) $\lim_{x \to \infty} (1 + e^{3x})^{1/(2x)}$
5. Suppose that \( f(4) = 11 \) and that \( f'(x) \geq 6 \) for all \( x \).

What does the Mean Value Theorem imply about the value of \( f(1) \)?

10 points

6. Amber wishes to fence a rectangular flower bed next to her house, which requires only 3 sides of fence (the other edge is the house itself). If the fence costs $3 per foot, and the bed is 200 \( ft^2 \), find the dimensions which minimize the total cost.

15 points

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7. A particle moves on the $x$-axis with acceleration $a(t) = 12 - 4t \text{ m/s}^2$ for $0 \leq t \leq 10$, with initial position $s(0) = 0$ and initial velocity $v(0) = -10 \text{ m/s}$.

Find the time period (if any) when the particle is moving to the right.

8. Find the most general antiderivatives:

(a) $f(t) = \frac{2t^3 - 3t^2 + 5t - 1}{\sqrt{t}}$

(b) $g(x) = \cos(x) + \sec(x) \tan(x) + \frac{1}{1+x^2} + \frac{1}{x} + \frac{1}{x^2}$