Calculus I : Summer 2009
Exam 1: July 2, 2009

INSTRUCTIONS : Show all of your work.

1. a) \[ \lim_{x \to -2} \left( \frac{3x^2 - 2}{2x^3 - 1} \right)^2 \]
   
   b) \[ \lim_{h \to 0} \frac{\sqrt{9 - 2h} - 3}{h} \]

   c) \[ \lim_{x \to -2^+} \frac{x + 2}{|x + 2|} \]

   d) \[ \lim_{x \to 2} \frac{x^2 + 3x - 10}{(x - 2)^3} \]

2. Let \( g(x) = \begin{cases} 
4x + 1 & \text{if } x < 1 \\
k & \text{if } 1 = x \\
2x^2 + 3 & \text{if } 1 < x < 2 \\
12 & \text{if } x = 2 \\
\frac{11}{2x - 3} & \text{if } 2 < x 
\end{cases} \)

   (a) Find the value of \( k \) (if possible) that makes \( g(x) \) continuous at \( x = 1 \).

   (b) Determine if \( g(x) \) is continuous at \( x = 2 \), or determine the type of discontinuity. If not continuous, is \( g(x) \) continuous from the right, left, or neither?
3. Prove that the equation \(2x - 1 = \cos(x)\) has at least one solution in the interval \([0, \pi]\).

4. Use the graph of \(f(x)\) to evaluate:

   (a) \(\lim_{x \to 2^-} f(x) = \)

   (b) \(\lim_{x \to 2^+} f(x) = \)

   (c) \(\lim_{x \to 2} f(x) = \)

   (d) \(\lim_{x \to 4^-} f(x) = \)

   (e) \(\lim_{x \to 4^+} f(x) = \)

   (f) \(\lim_{x \to 4} f(x) = \)

5. Use the definition to find \(h'(x)\), where \(h(x) = 1 + \frac{1}{x}\).
6. Find the equation of the tangent line to \( j(x) = 3x^3 - 4x \) when \( x = 2 \).

7. For \( t \geq 0 \) seconds, a particle has \( x \)-coordinate \( s(t) = -2t^3 - 3t^2 + 36t + 6 \) meters. Find the time(s) when the particle is moving to the right.
8. Suppose that \( f(x) \) and \( g(x) \) are differentiable functions, and \( f(2) = 3, \ g(2) = -1, \ f'(2) = 4, \ g'(2) = 1. \)

Evaluate

a) \((3f - 4g)'(2)\)  
b) \(\left(\frac{f}{1+fg}\right)'(2).\)

9. Differentiate the following functions (Do not simplify):

(a) \( y = \left(2x + \frac{2}{x^2} + \frac{1}{\sqrt{x}}\right)\left(x^4 - 19x^2 + 3x + 2\right).\)

(b) \( h(t) = \frac{2t^2 + 2t + 5}{t^2 + 6}\)