INSTRUCTIONS: Show all of your work, and give exact answers.

1. Evaluate the following limits exactly:

   (a) \( \lim_{x \to 3} \left( \frac{4x - 7}{x^2 + 1} \right)^3 \)

   (b) \( \lim_{h \to 0} \frac{\frac{1}{x + h} - \frac{1}{x}}{h} \)

   (c) \( \lim_{x \to 2^-} (x - |x|) \) (Note: \( |t| \) is the greatest integer function).

   (d) \( \lim_{x \to 1^-} \frac{3x^2 + 2x - 5}{(x - 1)^2} \)

   (e) \( \lim_{x \to -\infty} \left( \frac{\sqrt{5x^2 + 3x - 2}}{4x + 3} \right)^3 \)

   (f) \( \lim_{x \to -3^-} \frac{|x + 3|}{x + 3} \)
2. Let \( g(x) = \begin{cases} 
2x^2 + 5 & \text{if } x < 1 \\
 k & \text{if } x = 1 \\
9 - 2x & \text{if } 1 < x < 2 \\
5 & \text{if } x = 2 \\
\frac{24}{2x+1} & \text{if } 2 < x 
\end{cases} \)

(a) Find the value of \( k \) (if possible) which makes \( g(x) \) continuous at \( x = 1 \).

(b) Determine if \( g(x) \) is left-continuous, right-continuous, or continuous at \( x = 2 \). If it is not continuous, determine the type of discontinuity.

3. \textit{Prove} that the equation \( e^x = 4 - x \) has at least one solution in the interval \([0, 2]\).
4. The position (in meters) of an object after \( t \) seconds is given by \( s(t) = t^2 - 10t \).

(a) Use the limit definition to find the \emph{instantaneous velocity} \( v(t) \).

(b) Compare the instantaneous velocity at \( t = 3 \) to the \emph{average} velocity from \( t = 2 \) to \( t = 4 \).

5. Suppose that \( h'(x) = 3x^4 + 4e^{x-1} \) and \( h(1) = 5 \).

Write the \emph{equation} of the tangent line to \( y = h(x) \) at the point \( x = 1 \).
6. Use the graph of \( f(x) \) above to evaluate the limits:

\[
\begin{align*}
(a) \quad \lim_{x \to 2^-} f(x) &= \quad (b) \quad \lim_{x \to 2^+} f(x) &= \quad (c) \quad \lim_{x \to 2} f(x) = \\
(d) \quad \lim_{x \to 4^-} f(x) &= \quad (e) \quad \lim_{x \to 4^+} f(x) &= \quad (f) \quad \lim_{x \to 4} f(x) &=
\end{align*}
\]

7. Let \( f'(a) = \lim_{h \to 0} \frac{(2 + h)^2 e^2 \cdot e^h - 4e^2}{h} \)

Find \( f(x) \) and the value of \( a \).