1. (a) Give the precise definition of \( \lim_{x \to a} f(x) = L \).

5 points

(b) A manufacturer makes boxes with sides in the ratio of 3:2:1 (for example, 9x6x3 inches). If he wants to make a box of volume 48 cubic inches, with an allowable tolerance of 0.1 cubic inches, how close to 2 inches must the smallest side be? Give your answer to 2 decimal places.

5 points

(c) With reference to the definition of limit in 1a, and the measurements in 1b, what are \( x, f(x), a, L, \epsilon \) and \( \delta \)?

5 points

2. Let \( f(x) = \begin{cases} 4x^2 - 3 & \text{if } x < 2 \\ k & \text{if } x = 2 \\ 6x + 2 & \text{if } 2 < x \end{cases} \)

(a) Find all values of \( k \) for which \( f(x) \) is left-continuous at \( x = 2 \)

5 points

(b) Is it possible to choose \( k \) so that \( f(x) \) is continuous at \( x = 2 \)? If not, what kind of discontinuity does \( f(x) \) have?

5 points
3. (a) Give a precise statement of the Intermediate Value Theorem (IVT).

(b) Use the IVT to prove that the equation $x^3 - 3x = 7$ has a solution in the interval $(2,3)$.

4. Sketch the graph $y = f(x)$ of a function with the following properties:

- $f(x)$ is left-continuous at $x = 1$
- $f(x)$ has an infinite discontinuity at $x = 1$
- $f(x)$ is continuous at $x = 3$
- $f'(3)$ is not defined.

5. Use limits to find the derivatives of:

(a) $y = \sqrt{x}$

(b) $f(t) = \frac{1}{t}$
6. Find \( f(x) \) and \( a \) so that
\[
f'(a) = \lim_{h \to 0} \frac{(4 + h)^3 - 64}{h}
\]

7. Find the derivatives of the following functions. You do not need to simplify your answers.

(a) \( y = \frac{2}{3}x^{13/17} + \sqrt{x} - \frac{1}{x^4} + 4x^2 - e^3 \)

7. Find the derivatives of the following functions. You do not need to simplify your answers.

(b) \( f(t) = (t^3 - 2t^2 + 9)(t^4 + 4t) \)

(c) \( g(x) = \frac{5x^2 + 1}{4x - 3} \)

8. Find the equation of the tangent line to \( y = u^3 - 2u^2 + 5 \) when \( u = 1 \)
9. Assume that $f'(x)$ exists. Find the values of the following, in terms of $f'(x)$:

(a) \[ \frac{d}{dx} \left( \frac{f(x)}{x^3} \right) \]

(b) \[ D(\sqrt{x}f(x)) \]

10. Graham gently tosses a water balloon upwards out of an apartment window. Its height in feet $t$ seconds after it is thrown is given by $H(t) = 40 + 12t - 16t^2$.

(a) When does it hit the ground?

(b) What is the velocity at impact?

(c) What is the average velocity during its fall?