1. (10 pts) Evaluate $\int e^x \sin(5x) \, dx$

Folding. Let $u = \sin 5x$ and $dv = e^x \, dx$ so $du = 5 \cos 5x \, dx$ and $v = e^x$. Then $I = e^x \sin 5x - 5 \int e^x \cos 5x \, dx$. Let $u = \cos 5x$ and $dv = e^x \, dx$ so $du = -5 \sin 5x \, dx$ and $v = e^x$. The equation becomes $I = e^x \sin 5x - 5[e^x \cos 5x + 5I] = e^x \sin 5x - 5e^x \cos 5x - 25I$. Fold the $25I$ over to get $26I = e^x \sin 5x - 5e^x \cos 5x$, or $I = \frac{1}{26} (e^x \sin 5x - 5e^x \cos 5x)$.

2. (10 pts) Evaluate $\int \tan^3 x \sec^3 x \, dx$

The odd power of tangent forces you to write the integral as $\int \tan^2 x \sec^2 x (\sec x \tan x \, dx)$, so $u = \sec x$. Then $\tan^2 x = u^2 - 1$, so you get $I = \int (u^2 - 1)u^2 \, du = \int u^4 - u^2 \, du = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + c$. 