3450:223 Calculus III, Kreider  
**Quiz 9**, due on Wed 21 Oct, 14.7  
Name: ________________

This problem is worth 20 points.

1. Let \( f(x, y) = x^2 + 4xy + y^2 - 6y \).

   a) Find the critical points of \( f(x, y) \)
   \[
   f_x = 2x + 4y = 0 \quad \rightarrow \quad x = -2y \\
   f_y = 4x + 2y - 6 = 0 \quad \rightarrow \quad y = \frac{8y + 2y - 6}{-6} = 0; \quad y = -1 \quad \text{so} \quad x = 2 \\
   \text{CP: } (2, -1)
   \]

   b) Find the critical points of the trace \( f(x, 3) \)
   \[
   f(\mathbf{x}, 3) = x^2 + 12x + 9 - 18 = x^2 + 12x - 9 \\
   f_x = 2x + 12 = 0 \quad \rightarrow \quad x = -6
   \]

   c) Find the critical points of the trace \( f(x, -3) \)
   \[
   f(\mathbf{x}, -3) = x^2 - 12x + 9 + 18 = x^2 - 12x + 27 \\
   f_x = 2x - 12 = 0 \quad \rightarrow \quad x = 6
   \]

   d) Find the critical points of the trace \( f(3, y) \)
   \[
   f(3, y) = 9 + 12y + y^2 - 6y = y^2 + 6y + 9 \\
   f_y = 2y + 6 = 0 \quad \rightarrow \quad y = -3
   \]

   e) Find the critical points of the trace \( f(-3, y) \)
   \[
   f(-3, y) = 9 - 12y + y^2 - 6y = y^2 - 18y + 9 \\
   f_y = 2y - 18 = 0 \quad \rightarrow \quad y = 9
   \]

   f) Use the results of the previous problems to find the absolute max and absolute min of \( f(x, y) \) in the box \([-3, 3] \times [-3, 3] \).

   Candidates: (2, -1) and corners (CPs of traces are outside or a corner)

   \[
   f(2, -1) = 3 \\
   f(3, 3) = 36 \\
   f(-3, 3) = -36 \\
   f(3, -3) = 0 \\
   f(-3, -3) = 72
   \]

   abs max is 72 
   abs min is -36