Instructions Test #3 will be on Monday, April 18, 2005. The test covers the following sections in the text: §§4.1–4.5, 4.7, 4.9, 4.10. The following are a selection of problems from the material to be covered on the test. These problems do not represent the entirely of the types of problems you may appear on the test.

Solve these problems, ideally, without reference to your text. Solutions will be published on Thursday.

1. Find the absolute maximum of the function $f(x) = 18x + 15x^2 - 4x^3$ over the interval $[-3, 4]$

2. Find the critical numbers of each of the functions

   (a) $f(x) = \frac{x + 1}{x^2 + x + 1}$

   (b) $f(x) = x^{1/3}(x + 1)$
3. Find all critical points of the function \( f(x) = 3x^4 - 2x^2 + 1 \), and use the *Second Derivative Test* to decide which gives a local maximum, and which gives a local minimum. (Show enough detail to prove to me that you are indeed applying the second derivative test.)

4. Consider the function \( f(x) = x^{2/3} (x + 1) \), it is easy to compute \( f'(x) = \frac{5x + 2}{3x^{1/3}} \). Using first derivative methods, find the interval(s) of *increase* and the interval(s) of *decrease*. Identify the critical numbers and classify them as *local maximums* or *local minimums*.

5. Find the *points of inflection* and the *intervals of concavity* for the function \( f(x) = 3x^4 - 2x^2 + 1 \).
6. Consider the function \( y = \frac{x^2}{x^2 - 4} \); it is easy to calculate \( y' = -\frac{8x}{(x^2 - 4)^2} \). Identify any horizontal and vertical asymptotes, and using first derivative information, make a rough sketch of the graph of the function.

7. Compute each of the limits at infinity.
   (a) \( \lim_{x \to \infty} \frac{4x - 5x^3}{2x^3 - 2x^2 + 1} \)
   
   (b) \( \lim_{x \to \infty} (\sqrt{9x^2 + x} - 3x) \)

8. Find the dimensions of a rectangle with an area of 1000 m\(^2\) whose perimeter is a small as possible.
9. If 2700 cm\(^2\) of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

10. Find the most general antiderivative of the given function.
   (a) \(g(x) = 2x \sqrt{x} - \frac{6}{x^{4/7}}\)  
   (b) \(h(x) = 6 \sin(x) - 2 \csc^2(x)\)

11. Suppose \(f'(x) = 5x^9 - \frac{3}{x^3}\) and \(f(1) = 4\). Find \(f\).

12. We want to solve the equation \(x^3 = 4\) numerically using Newton’s Method. First set-up the Newton’s Iteration Formula for this equation. Using the iteration formula and an initial guess of \(x_0 = 2\), calculate the next estimate, \(x_1\).