Instructions: (10 points) Use Newton’s Method to find the solution to the given equation. Follow the step-by-step outline that accompanies this assignment. Put your results into a table similar to the one attached to these problems. Do enough iterations until \( f(x_n) = 0.000000 \), again, see the accompanying example. When drawing a table, use a straight edge. Be neat.

Note: Show 6 decimal places for the numbers entered into your tables.

1. Consider the equation: \( x^4 + x = 3 \). There are two solutions \( (r_1 \text{ and } r_2) \) to this equation: \(-2 \leq r_1 \leq -1\) and \( 1 \leq r_2 \leq 2 \).
   (a) Find \( r_1 \) using an initial guess of \( x_1 = -2 \).
   (b) Find \( r_2 \) using an initial guess of \( x_1 = 1 \).
**Problem:** Let $f$ be a differentiable function, and consider the equation $f(x) = 0$. We want to solve this equation numerically for $x$.

**Solution:** Proceed as follows:

**Step 1:** Compute $f'(x)$.

**Step 2:** Construct Newton’s Iteration Formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, 3, \ldots$$  (1)

**Step 3:** Construct a table of estimates. Choose an initial estimate $x_1$, and use your Newton Iteration Formula, equation (1), to compute successively more accurate estimates of the unknown value.

1. Make an initial guess of $x_1$. For $n = 1$, the equation (1) becomes $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. The value of the right-hand side depends on $x_1$, which is known, so $x_2$ can be computed. Thus, $x_2$ is now known.

2. For $n = 2$, the equation (1) becomes $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$. The value of the right-hand side depends on $x_2$, which is known, so $x_3$ can be computed. Thus, $x_3$ is now known.

3. And so on for $n = 3, 4, 5, \ldots$

The example below illustrates a complete solution, and proper tabulation of the results.

**Example 1** Find the positive root of the equation $x^2 = 2$.

**Solution:** The function is $f(x) = x^2 - 2$.

**Step 1:** Compute derivative, $f'(x) = 2x$.

**Step 2:** Construct the iteration formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{x_n^2 + 2}{2x_n}$$

Thus, for this problem, the iteration formula is

$$x_{n+1} = \frac{x_n^2 + 2}{2x_n}$$

This, together with an initial guess of $x_1 = 1.5$ yields the following calculations.

**Step 3:** Construct a table of estimates.

Initial guess of $x_1 = 1.5$ and iteration formula of $x_{n+1} = \frac{x_n^2 + 2}{2x_n}$.

<table>
<thead>
<tr>
<th>Newton’s Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2 - 2$, $x_1 = 1.5$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
</tr>
</tbody>
</table>

Thus, the positive root to the equation $x^2 - 2 = 0$ is $x \approx 1.4142135$ or, $\sqrt{2} \approx 1.4142135$. 