Instructions: (100 points) Solve each problem and box in your final answer.

1. Fill in the blank to complete each of the following statements.
   (a) **Statistics** is the science of collecting, organizing, analyzing and interpreting data in order to make **decisions**.
   (b) **Descriptive** statistics is the branch of statistics that involves the organization, summarization, and display of data.
   (c) If the probability of passing this course is 0.85, then the probability of not passing is 0.15.
   (d) Two events $A$ and $B$ are **independent** if either $P(A|B) = P(A)$ or $P(B|A) = P(B)$.

2. Respond to each of the following statements by inserting a 'T' for true, or a 'F' for false, in the blank space provided.
   (a) **F** Every data set must have a mode.
   (b) **T** The first quartile, $Q_1$, partitions a data set so that 25% of the data are less than it, and 75% of the data are greater than it.
   (c) **T** A z-score of $z = 1.2$ means the data point is 1.2 standard deviation units above the mean.
   (d) **F** For the probability experiment of rolling a fair six-sided die, the event $E$ of rolling an even number is a simple event.

3. Complete the table below by filling in all the columns. Also, compute the total size, $n$, of the data set and the grouped mean $\bar{x}$.

<table>
<thead>
<tr>
<th>Class</th>
<th>Frequency $f$</th>
<th>Midpoint $x$</th>
<th>$f \cdot x$</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5 – 18.5</td>
<td>6</td>
<td>12.5</td>
<td>75.0</td>
<td>6</td>
</tr>
<tr>
<td>18.5 – 30.5</td>
<td>10</td>
<td>24.5</td>
<td>245.0</td>
<td>16</td>
</tr>
<tr>
<td>30.5 – 42.5</td>
<td>13</td>
<td>36.5</td>
<td>474.5</td>
<td>29</td>
</tr>
<tr>
<td>42.5 – 54.5</td>
<td>8</td>
<td>48.5</td>
<td>388.0</td>
<td>37</td>
</tr>
<tr>
<td>Totals</td>
<td>37</td>
<td></td>
<td>1182.5</td>
<td></td>
</tr>
</tbody>
</table>

Data size: $n = 37$  
Mean: $\bar{x} = 31.96 \approx 32$

4. The time in seconds for a sample of six sports cars to go from 0 to 60 mph is given below:

$3.7 \ 4.0 \ 4.4 \ 4.8 \ 4.8 \ 5.1$

Compute each of the statistics.
(a) (3 pts) The mean: $\bar{x} = 4.47$
(b) (3 pts) The median: Median = 4.6
(c) (2 pts) The mode: Mode = 4.8
(d) (4 pts) The (sample) variance: $s^2 = 0.2867$
5. Consider a Box-and-Whisker Plot.

\[ x \]

\[ -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14 \quad 16 \]

(a) Approximate the first quartile: \( Q_1 = 5 \)
(b) Approximate the interquartile range: \( \text{IQR} = 11 - 5 = 6 \)
(c) Approximate the range of the data: \( \text{range} = 15 - 1 = 14 \)

6. A Company offers starting salaries with \( \mu = $35,000 \) with \( \sigma = $5,000 \). Use the Empirical Rule to approximate the percentage of new hires that are offered a starting salary over $40,000. (Assume the distribution of starting salaries is bell-shaped.)

Answer = 16%

Solution: We compute the area of the region under the standard bell-shaped curve beyond one standard deviation above the mean. That calculation is \( 0.5 - 0.34 = 0.16 \). (This is the area under the half above the mean, minus the area within one standard deviation above the mean \( 0.68/2 = 0.34 \).)

7. A student takes two exams, one in statistics and the other in biology. The mean and standard deviation for the statistics exam was 63 and 7.0, respectively; the mean and standard deviation for the biology exam was 26 and 3.9, respectively. The student scored a 73 on the statistics exam and a 26 on the biology exam.

(a) (5 pts) Transform each test score to a z-score.

\[ z\text{-score for statistics class} = 1.43 \]
\[ z\text{-score for biology class} = 0 \]

Solution: The calculations are

\[ z\text{-score for statistics class} = \frac{73 - 63}{7} = 1.43 \]
\[ z\text{-score for biology class} = \frac{26 - 26}{3.9} = 0 \]

(b) (3 pts) Determine on which test the student had the better score. Explain.

Solution: The student did better on the statistics exam because s/he was one standard deviation above the mean, whereas on the biology test, s/he scored exactly the mean. In symbols,

\[ z\text{-score for statistics class} = 1 > 0 = z\text{-score for biology class} \]

so s/he did better on the stat test.
8. Two radar sites act independently of each other. For each, the probability a station detects an intruder into restricted air space is 0.90.

\[ R_1 = \text{event station 1 detects intruder} \quad P(R_1) = 0.90 \]
\[ R_2 = \text{event station 2 detects intruder} \quad P(R_2) = 0.90 \]

(a) (3 pts) Using the independence assumption, compute \( P(R_1 \text{ and } R_2) = 0.81 \).

\[ P(R_1 \text{ and } R_2) = P(R_1) \cdot P(R_2) = 0.90 \cdot 0.90 = 0.81 \]

(b) (2 pts) Using the addition rule and (a), find \( P(R_1 \text{ or } R_2) = 0.99 \).

\[ P(R_1 \text{ or } R_2) = P(R_1) \cdot P(R_2) - P(R_1 \text{ and } R_2) \\
= 0.90 + 0.90 - 0.81 = 0.99 \]

and that is why the sky is covered by multiple radar stations.

9. An experiment consists of flipping three coins, a nickel, a dime and a quarter. The outcome of this experiment is represented as a sequence of Hs and Ts; example, a particular outcome is represented as THT to indicate the results of the nickel, dime and quarter, respectively.

(a) (3 pts) List the sample space for this experiment

\[ S = \{ TTT, TTH, THT, HTT, HHT, HTH, THH, HHH \} \]

(b) (3 pts) Compute the probability of flipping two heads.

\[ P(\text{two heads}) = \frac{3}{8} = 0.375 \]

(c) (3 pts) Compute the probability of flipping at least one head.

\[ P(\text{at least one head}) = \frac{7}{8} = 0.875 \]

(d) (2 pts) Compute the probability of flipping a head or a tail. \( P(\text{a head or a tail}) = 1 \)

10. In a box of 11 parts, four of the parts are defective. Two parts are selected—one at a time—at random without replacement. Find the probability that both parts are defective.

\[ P(\text{defective on first selection & defective on second selection}) = \frac{4}{11} \cdot \frac{3}{10} \approx 0.11 \]
11. The table shows the results of a survey in which 142 men and 145 women workers ages 25 to 64 were asked if they have at least one month's income set aside for emergencies.

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than one month’s income</td>
<td>66</td>
<td>83</td>
<td>149</td>
</tr>
<tr>
<td>At least one month’s income</td>
<td>76</td>
<td>62</td>
<td>138</td>
</tr>
<tr>
<td>Total</td>
<td>142</td>
<td>145</td>
<td>287</td>
</tr>
</tbody>
</table>

We now randomly select a worker from the 287 workers in the survey. For the parts that follow, we define the events:

- \( F \) = the event worker is female
- \( M \) = the event worker is male
- \( L \) = the event worker has less than one month’s income
- \( A \) = the event that the worker has at least one month’s income

In parts (a) – (d), show calculations on the back of page 3.

(a) Find the probability that the selected worker has at least one month’s income. In symbols, find \( P(A) \).

\[
P(A) = \frac{138}{287} \approx 0.48
\]

(b) Given that selected worker is a male, find the probability the worker has less than one month’s income. In symbols, find \( P(L|M) \).

\[
P(L|M) = \frac{66}{142} \approx 0.46
\]

(c) Given that the selected worker has at least one month’s income, find the probability the worker is female. In symbols, find \( P(F|A) \).

\[
P(F|A) = \frac{62}{138} \approx 0.45
\]

(d) Find the probability that the selected worker is either a female or has at least one month’s income set aside for emergencies. In symbols, find \( P(F \text{ or } A) \).

\[
P(F \text{ or } A) = \frac{138}{287} + \frac{145}{287} - \frac{62}{287} = \frac{221}{287} \approx 0.77
\]

Solution: We use the Addition Rule.

\[
P(F \text{ or } A) = P(F) + P(A) - P(F \text{ and } A)
\]

\[
= \frac{138}{287} + \frac{145}{287} - \frac{62}{287} = \frac{221}{287} \approx 0.77
\]