Instructions The final exam will be on Monday, December 13, 2004, 8:00–9:55 am, in a room yet to be determined. This is a two-part final exam: The first part is basically test #4 (§§5.1–5.5; 6.1–6.2) for which this document is a review; the second part is a review of the semester’s material (§§1.1–4.10), use the previous reviews and tests as well as your text and notes as resource documents.

The following is a selection of problems from the material to be covered on the first part of the final exam, which covers §§5.1–5.5, 6.1–6.2. These problems do not represent the entirely of the types of problems you may appear on the test. Solve these problems, ideally, without reference to your text.

1. State the Fundamental Theorem of Calculus, Part I.
2. Differentiate each of the following function.
   (a) \( g(x) = \int_{-1}^{x} \sin(t^2) \, dt \)
   (b) \( g(x) = \int_{x}^{3} \sqrt{t^2 + 1} \, dt \)
   (c) \( g(x) = \int_{0}^{3x^2} \tan(4t) \, dt \)
   (d) \( g(x) = \int_{2 \sqrt{x}}^{\sqrt{x^2 + 1}} dt \)

3. Elementary Integration. Evaluate each of the following definite and indefinite integrals.
   (a) \( \int_{1}^{2} 6x^2 \, dx \)
   (b) \( \int_{-\pi/2}^{\pi/2} \cos(x) \, dx \)
   (c) \( \int 7t^3 - 4 \sin(t) + 3 \csc^2(t) \, dt \)
   (d) \( \int_{-1}^{2} |x - 1| \, dx \)

4. Integration. Evaluate each of the following definite and indefinite integrals.
   (a) \( \int \cos(3x) \, dx \)
   (b) \( \int x^2(4x^3 + 1)^{3/2} \, dx \)
   (c) \( \int \frac{4}{(3z + 1)^3} \, dz \)
   (d) \( \int_{0}^{3} \sqrt{x + 1} \, dx \)
   (e) \( \int \sec^2(5\theta) \, d\theta \)

5. Find the area of the region above the x-axis, under the graph of \( f(x) = x^2 + 1 \), between \( x = 1 \) and \( x = 2 \).

6. Find the area between the two curves \( f(x) = x^2 \) and \( g(x) = 8 - x^2 \).

7. Consider the region bounded by the two intersecting curves \( y = x \) and \( x = y^3 \).
   (a) Set up the integral using \( x \) as the variable of integration.
   (b) Set up the integral using \( y \) as the variable of integration.
8. Consider the region bounded by the $x$-axis, the graph $y = 2x^2$, and the lines $x = 0$ and $x = 1$.
   
   (a) Rotate the region around the $x$ axis, find the volume of the generated solid.
   
   (b) Rotate the region around the $y$ axis, set up the volume integral of the generated solid.

9. A solid $S$ has a semi-circular base bounded by the $x$ axis and the graph of $y = \sqrt{1 - x^2}$. Each cross section perpendicular to the $x$ axis is a square. Find the volume of the solid.