Instructions Test #3 will be on Tuesday, November 16, 2004. The test covers the following sections in the text: §§4.1–4.5, 4.7, 4.9, 4.10. The following are a selection of problems from the material to be covered on the test. These problems do not represent the entirely of the types of problems you may appear on the test.

Solve these problems, ideally, without reference to your text. Solve the problems first, then look at the solutions.

1. Find the absolute maximum of the function \( f(x) = 18x + 15x^2 - 4x^3 \) over the interval \([-3, 4]\)

2. Find the critical numbers of each of the functions
   
   (a) \( f(x) = \frac{x + 1}{x^2 + x + 1} \)

   (b) \( f(x) = x^{1/3}(x + 1) \)

3. Find all critical points of the function \( f(x) = 3x^4 - 2x^2 + 1 \), and use the Second Derivative Test to decide which gives a local maximum, and which gives a local minimum. (Show enough detail to prove to me that you are indeed
4. Consider the function \( f(x) = x^{2/3}(x + 1) \), it is easy to compute \( f'(x) = \frac{5x + 2}{3x^{1/3}} \). Using first derivative methods, find the interval(s) of increase and the interval(s) of decrease. Identify the critical numbers and classify them as local maximums or local minimums.

5. Find the points of inflection and the intervals of concavity for the function \( f(x) = 3x^4 - 2x^2 + 1 \).

6. Consider the function \( y = \frac{x^2}{x^2 - 4} \); it is easy to calculate \( y' = -\frac{8x}{(x^2 - 4)^2} \). Identify any horizontal and vertical
asymptotes, and using first derivative information, make a rough sketch of the graph of the function.

7. Compute each of the limits at infinity.
   (a) \( \lim_{x \to \infty} \frac{4x - 5x^3}{2x^3 - 2x^2 + 1} \)

   (b) \( \lim_{x \to \infty} (\sqrt{9x^2 + x - 3x}) \)

8. Find the dimensions of a rectangle with an area of 1000 m\(^2\) whose perimeter is a small as possible.

9. If 1200 cm\(^2\) of material is available to make a box with a square base and an open top, find the largest possible
volume of the box.

10. Find the most general antiderivative of the given function.
   (a) \( g(x) = 2x \sqrt{x} - \frac{6}{x^{4/7}} \)  
   (b) \( h(x) = 6 \sin(x) - 2 \csc^2(x) \)

11. Suppose \( f'(x) = 5x^9 - \frac{3}{x^3} \) and \( f(1) = 4 \). Find \( f \).

12. We want to solve the equation \( x^3 = 4 \) numerically using Newton’s Method. First set-up the Newton’s Iteration Formula for this equation. Using the iteration formula and an initial guess of \( x_0 = 2 \), calculate the next estimate, \( x_1 \).