Instructions: Use Newton’s Method to find the solution to the given equation. Follow the step-by-step outline that accompanies this assignment. Put your results into a table similar to the one attached to these problems. Do enough iterations until $f(x_n) = 0.000000$, again, see the accompanying example. When drawing a table, use a straight edge. Be neat.

Note: Show 6 decimal places for the numbers entered into your tables.

1. Depending on your last name, solve the equation using Newton’s method. Use as your initial guess $x_0 = .5$. Important: When dealing with trigonometric functions, be sure your calculator is in radian mode.

   If your last name begins with...

   (a) A–H, find the solution to the equation $\cos(3x/2) = x$ between $0 < x \leq 1$.
   (b) I–P, find the solution to the equation $\cos(5x/2) = x$ between $0 < x \leq 1$.
   (c) Q–Z, find the solution to the equation $\cos(7x/2) = x$ between $0 < x \leq 1$. 

**Problem:** Let \( f \) be a differentiable function, and consider the equation \( f(x) = 0 \). We want to solve this equation numerically for \( x \).

**Solution:** Proceed as follows:

**Step 1:** Compute \( f'(x) \).

**Step 2:** Construct Newton’s Iteration Formula:

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, 3, \ldots
\]  

(1)

**Step 3:** Construct a table of estimates. Choose an initial estimate \( x_0 \), and use your Newton Iteration Formula, equation (1), to compute successively more accurate estimates of the unknown value.

1. Make an initial guess of \( x_0 \). For \( n = 0 \), the equation (1) becomes \( x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \). The value of the left-hand side depends on \( x_0 \), which is known, so \( x_1 \) can be computed. Thus, \( x_1 \) is now known.

2. For \( n = 1 \), the equation (1) becomes \( x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \). The value of the left-hand side depends on \( x_1 \), which is known, so \( x_2 \) can be computed. Thus, \( x_2 \) is now known.

3. And so on for \( n = 2, 3, 4, \ldots \)

The example below illustrates a complete solution, and proper tabulation of the results.

**Example 1** Find the positive root of the equation \( x^2 = 2 \).

**Solution:** The function is \( f(x) = x^2 - 2 \).

**Step 1:** Compute derivative, \( f'(x) = 2x \).

**Step 2:** Construct the iteration formula:

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{x_n^2 + 2}{2x_n}
\]

Thus, for this problem, the iteration formula is

\[
x_{n+1} = \frac{x_n^2 + 2}{2x_n}
\]

This, together with an initial guess of \( x_0 = 1.5 \) yields the following calculations.

**Step 3:** Construct a table of estimates.

<table>
<thead>
<tr>
<th>Newton’s Method</th>
<th>( f(x) = x^2 - 2 ), ( x_0 = 1.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( x_n )</td>
</tr>
<tr>
<td>0</td>
<td>1.50000000</td>
</tr>
<tr>
<td>1</td>
<td>1.41666667</td>
</tr>
<tr>
<td>2</td>
<td>1.41421568</td>
</tr>
<tr>
<td>3</td>
<td>1.41421356</td>
</tr>
<tr>
<td>4</td>
<td>1.41421356</td>
</tr>
</tbody>
</table>

Thus, the positive root to the equation \( x^2 - 2 = 0 \) is \( x \approx 1.4142135 \) or, stated differently, \( \sqrt{2} \approx 1.4142135 \).