Potential project topics for Math 636

These are some potential project topics for this semester for the required project. I will be adding to this list throughout the semester, but this is what we have so far. Remember, you may work alone or in pairs, and you need both a presentation (about 20-25 minutes) and a short (4-5 page) paper about your topic. Also, this list is far from exhaustive. Any topic in combinatorics or graph theory is fair game, as long as it is not something we directly covered in class. Any topic needs to be cleared with me first, however, so I can make sure it is appropriate and that somebody else has not already claimed it.

(1) In class we proved that $c_{p,k} = b_{p,k}$ through a very roundabout method - by constructing a bijection from the “good” paths to the parenthese operations to the polygon dissections to the $k$-ary trees. IS there a more direct way to do this? Can you construct an explicit bijection between the two sets that does not use all the intermediary sets?

(2) In class we discussed the colorings of the corners of a cube. But the cube is just one of several “Platonic” solids - including the octahedron, tetrahedron, dodecahedron, and icosahedron. What are the symmetry groups of each one, and for each $k$, how many ways as there to $k$-color each shape? What about the Archimedean solids? What if we color edges or faces instead of corners?

(3) Polya enumeration can be used to count the number of graphs with a given number of vertices, by coloring each edge either "visible" or "invisible". To count the number of truly distinct (i.e. non-isomorphic) graphs with $n$ vertices, we have to have the symmetric group $S_n$ act on the set of graphs. How does this work with Polya enumeration, and demonstrate how to count the number of distinct graphs with small numbers of vertices using this method.

(4) There are literally hundreds of equivalent characterizations of the Catalan numbers (as seen in Stanley’s collection, which is online somewhere). Many, if not all, of these have natural generalizations, like we saw with the paths and the trees and the polygon dissections. Discuss some of the natural generalizations of the other Catalan objects, and show whether they are or are not equivalent to the ones we were looking at in class.

(5) The theory of Burmann-Lagrange series in complex analysis allows one to explicitly compute the coefficients of the generating function for the generalizations of the Catalan numbers from the expression

$$x(h(x))^p - h(x) + 1 = 0$$

that we saw in class. Explain how this works.
(6) Determine a formula for $T_{p,k}$, the number of distinct dissections of a regular $((p - 1)k + 2)$-gon into $k p + 1$-gons. Two of my previous students made progress on this problem in their theses, you can either summarize their results or try to find your own.

(7) There are likely about a gajillion (my estimate may be imprecise) variations and generalizations of the coin-weighing problem we discussed in class. Discuss some non-trivial versions of this.

(8) Give a proof of Kruskal’s algorithm. Discuss the connection between Kruskal’s algorithm and Prim’s algorithm. Do they always yield the same spanning trees? Give some non-trivial applications of these algorithms.

(9) In class we discussed (or will discuss, depending on when you’re reading this!) the "quick tour" solution to the traveling salesman problem, and mentioned that under certain conditions the solution given by the quick tour will not be too far off from the optimal solution. The book (Tucker) gives a half-hearted proof of this, but I want to see a real proof, and a discussion of when the quick tour gives the true minimal tour.