Math 223, Section 5
Exam I
4/1/16

Name: _______________________________________

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**NOTE:** I need to see all of your work for each problem. Unjustified work will receive little or no credit.
1. (20 points) Test each of the following series for convergence. If you are using a particular test, be sure to show that the conditions of the test are met.

(a) (7 points)

\[ \sum_{n=1}^{\infty} \frac{1}{n \sqrt{\ln n}}. \]

(b) (7 points)

\[ \sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}. \]

(c) (6 points)

\[ \sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}. \]
2. (20 points) Determine if each of the following series is absolutely convergent, conditionally convergent, or neither. Be sure to justify your answer.

(a) (7 points)
\[ \sum_{n=1}^{\infty} (-1)^{n-1} \frac{5n^2}{6n^3 + 1}. \]

(b) (7 points)
\[ \sum_{n=1}^{\infty} \frac{n^{2n}}{(1 + 2n^2)^n}. \]

(c) (6 points) Determine whether
\[ \sum_{n=1}^{\infty} \frac{\cos 5n}{n^3 + 5n^2 + 2n + 7} \]
is absolutely convergent. (Do not worry about conditional convergence for this one.)
3. (20 points) Evaluate each of the following convergent series:

(a) (5 points)
\[
\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n-1}.
\]

(b) (5 points)
\[
\sum_{n=1}^{\infty} \left[6 \left(\frac{2}{3}\right)^n - 3 \left(\frac{4}{5}\right)^{n-1}\right].
\]

(c) (5 points)
\[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots\]

(d) (5 points)
\[
\sum_{n=1}^{\infty} \frac{1}{n(n+1)}.
\]
(Hint: partial fractions are your friend here.)
4. (20 points) Compute the surface area of the solid formed by rotating 
\( y = \sqrt{1 + e^x} \), with \( 0 \leq x \leq 1 \), around the \( x \)-axis. (Hint: it looks messy, but if you do the right algebra the integral will clean up to something very nice!)
5. (20 points) Compute the following improper integral or show it diverges:

\[ \int_{1}^{\infty} \frac{\ln x}{x^4} \, dx. \]