Inscribed (Cyclic) Quadrilaterals and Parallelograms

Lesson Summary:
This lesson introduces students to the properties and relationships of inscribed quadrilaterals and parallelograms. Inscribed quadrilaterals are also called cyclic quadrilaterals. These relationships are:
1. If a quadrilateral is inscribed inside of a circle, then the opposite angles are supplementary.
2. If a parallelogram is inscribed inside of a circle, it must be a rectangle.

In the Extension Activities students are introduced to Ptolemy’s Theorem and maltitudes.

Key Words:
Inscribed, cyclic quadrilaterals, parallelogram, Ptolemy’s Theorem, maltitudes

Existing Knowledge
These above relationships are normally taught in a chapter concerning circles. In order to answer the extensions questions, students need to be knowledgeable of the arc-angle relationships of a circle and properties of parallelograms.

NCTM/State Standards
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. (NCTM)

Learning Objectives
Students will be able to calculate angle and arc measure given a quadrilateral inscribed in a circle.

Students will be able to identify a rectangle as the only parallelogram that can be inscribed in a circle.

Materials
Cabri II or Geometer’s Sketchpad

Procedure/Review
The students may need a review concerning arc-angle relationships of a circle (central angles, inscribed angles, angles in the interior and exterior of a circle) and properties of parallelograms.

Students should be paired in groups of two by teacher’s method of choice. To introduce the lesson, the teacher may want to pose the question, “Can a parallelogram be inscribed inside of a circle?”

The teacher may assess the students based on the application questions at the end of the lesson.
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Group Members’ Names: ___________________________________________________

___________________________________________________

File Name: ______________________________________________________________

Activity Goals

In this activity you and your partner will answer the following question, "Can a parallelogram be inscribed in a circle?" You and your partner will also determine relationships among the angles of an inscribed quadrilateral.

Open a new page in Cabri II and follow the instructions below.

Laboratory One

1. Draw a circle and label the center O. [Circle Tool]
2. Draw a segment whose endpoints lie on the circle and label the points A and B. [Segment Tool]
3. Place a point anywhere on the circle and label it C. See diagram below. [Point Tool]
4. Construct a line parallel to segment AB through point C and label this line l. [Construct Tool and Label Tool]
5. Create a point at the intersection of the circle and the line l. Label this point as D. [Point Tool]
6. Construct a segment CD. [Segment Tool]
7. Hide line l. [Hide Tool]
8. Construct a segment between points C and B. [Segment Tool]
9. Construct a line parallel to segment BC through point D. [Construct Tool]

Answer the following questions.

10. Is the quadrilateral inside the circle a parallelogram? How do you know?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

[Diagram of a circle with labeled points A, B, C, D, and O. A quadrilateral is inscribed in the circle.]
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11. Is this parallelogram inscribed in the circle? Why or why not?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

12. Grab point B and move it around the circle. Can you make the parallelogram become inscribed in the circle?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

13. Grab point C. Can you make the parallelogram become inscribed in the circle?
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Using the Point Tool, create an intersection point between line l and segment AB.

Move point C until the newly created intersection point coincides with point A.

14. Measure each angle and record the measures in the chart below. [Measure Tool]

<table>
<thead>
<tr>
<th>&lt;A</th>
<th>&lt;B</th>
<th>&lt;C</th>
<th>&lt;D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15. Are your angles close to 90°? __________
16. What type of parallelogram has four right angles?
________________________________________________________________________
________________________________________________________________________

17. What is the only type of parallelogram that can be inscribed in a circle?
________________________________________________________________________
________________________________________________________________________
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Laboratory Two
1. Draw a circle and label the center O. [Circle Tool]
2. Draw four segments connecting them to form a quadrilateral. [Segment Tool]
3. Label these points A, B, C, and D respectively. [Label Tool]

4. Measure each angle and record the measures in the chart below. [Measure Tool]

<table>
<thead>
<tr>
<th>&lt;A</th>
<th>&lt;B</th>
<th>&lt;C</th>
<th>&lt;D</th>
<th>&lt;A + &lt;C</th>
<th>&lt;D + &lt;B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Add <D and <B. [Calculate Tool]
6. Drag the result onto the work area.
7. Delete the word “Result:” and type “<D+<B”. [Comment Tool]
8. Add <C and <A. [Calculate Tool]
9. Drag the result onto the work area.
10. Delete the word “Result:” and type “<C+<A”. [Comment Tool]
11. What do each of these sums equal? ____________________
12. Grab and move each of the vertices around the circle. Do the sums you determined in #11 change? _______________
13. State the relationship between the opposite angles of an inscribed quadrilateral.
   ________________________________________________________________
   ________________________________________________________________
   ________________________________________________________________
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Extension Laboratory One

In this activity you and your partner will discover another interesting property of inscribed quadrilaterals.

1. Draw a circle and label it O. [Circle Tool]
2. Draw four segments connecting them to form a quadrilateral. [Segment Tool]
3. Label these points A, B, C, and D respectively. [Label Tool]
4. Draw the diagonals of the quadrilateral. [Segment Tool]

5. Measure each side of the quadrilateral and each diagonal. [Measure Tool]
6. Find the products of each pair of opposite sides. Record your answers in the table below. [Calculate Tool]
7. Find the product of the diagonals. Record your answer in the table below. [Calculate Tool]
8. Add the two products of each pair of opposite sides. Record your answer in the table below. [Calculate Tool]

<table>
<thead>
<tr>
<th>DC*AB</th>
<th>DA*CB</th>
<th>CA*DB</th>
<th>(DC<em>AB)+( DA</em>CB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. What do you notice about the product of the diagonals and the sum of the products of each pair of opposite sides?
_____________________________________________________________________
_____________________________________________________________________

10. Grab and move a vertex of the quadrilateral. Does this relationship change? ______
11. Grab and move a vertex past the non-moving diagonal. Why does this relationship not work?
_____________________________________________________________________
_____________________________________________________________________

This relationship is called Ptolemy’s Theorem in honor of Alexandrian mathematician Claudius Ptolemaeus (100-168 A.D.). This theorem appeared in Ptolemy’s book, *Almagest*, which discussed his observations about astronomy.
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Extension Laboratory Two

In a triangle, the altitude starts at a vertex and is perpendicular to the opposite side. Quadrilaterals have a similar construction. The malitude of a quadrilateral passes through the midpoint of a side and is perpendicular to the opposite side.

1. Draw a circle and label it O. [Circle Tool]
2. Draw four segments connecting them to form a quadrilateral. [Segment Tool]
3. Label these points A, B, C, and D respectively. [Label Tool]
4. Find the midpoints of each side. [Midpoint Tool]
5. Label these points W, X, Y, and Z respectively. [Label Tool]
6. Construct a line through point W that is perpendicular to segment AB. [Construct Tool]

7. Construct a line through point X that is perpendicular to segment BC. [Construct Tool]
8. Construct a line through point Y that is perpendicular to segment DC. [Construct Tool]
9. Construct a line through point Z that is perpendicular to segment AD. [Construct Tool]
10. State relationship do you see about the malitudes of an inscribed quadrilateral.

_____________________________________________________________________
_____________________________________________________________________
_____________________________________________________________________
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Application Questions

1. Given that an angle whose vertex lies on a circle is one-half its intercepted arc, use the diagram to the right to show that the opposite angles of an inscribed quadrilateral are supplementary.

   \[ m<A = \text{__________} \]
   \[ m<B = \text{__________} \]
   \[ m<C = \text{__________} \]
   \[ m<D = \text{__________} \]

2. Using the diagram to the right, find the measure of \(<A, <B, <C, \text{ and } <D.

   \[ m<A = \text{__________} \]
   \[ m<B = \text{__________} \]
   \[ m<C = \text{__________} \]
   \[ m<D = \text{__________} \]

3. In Activity One you learned that a rectangle was the only parallelogram that can be inscribed in a circle. What is the only type of rhombus that can be inscribed in a circle?

   \[ \text{__________} \]
   \[ \text{__________} \]
   \[ \text{__________} \]
   \[ \text{__________} \]
   \[ \text{__________} \]
   \[ \text{__________} \]

4. The angle bisectors of a triangle meet at a point called the incenter. The angle bisectors of any quadrilateral sometimes meet in a point and sometimes do not meet in a point. When the angle bisectors of any quadrilateral do not meet in a point, what type of quadrilateral do they form? (Hint: These points may encircle you.)

   \[ \text{__________} \]