Napoleon Triangle

Lesson Summary:
The students will be placed in pairs to help them learn to cooperate and help one another through self-discovery and the cooperative activity. The students will work individually on journal prompts. This will help develop creativity and written communication skills.

Key Words:
Triangle, modern geometry, napoleon triangle

Background Knowledge:
- Students will have prior lessons on triangles and equilateral triangles.
- Students will have prior experience in recognizing patterns.
- Students must have some prior geometry and problem solving skills.
This information helps the teacher to plan the daily lesson prior to this activity to ensure that students have this prior knowledge. It will be important to know that the students can recognize patterns.

Learning Objectives:
- Students will discover the relationship between the boundary points, interior points, and area of lattice point polygons – Pick’s Theorem.
- Self discovery will help the students become self-learners.
- Students will collaborate with others.

Materials:
This lesson can be done one of two ways. The students can either work together on Cabri Geometry II software on computers or calculators. Or the students can use Geo Boards.
- Overhead to give students information
- Geo Boards or Computers/Calculators equipped with Cabri Geometry II.
- Lab Handout
- Journal Prompt

Suggested Procedure:
This lesson is designed for a Math class consisting of 10th and 11th grade students in Geometry. The time allocated will be one day with a take home assignment.

Grouping:
Students will be put in pairs for the self discovery and cooperative activity. They will take notes and observe individually during whole group instruction. Students will also have an individual take home journal prompt.

Students with Special Needs:
Special Needs students will be grouped with others and will be able to participate in group activities and peer collaboration. Available equipment will be used if students are visually or hearing impaired. Low level students may be given modified questions to answer in addition to group activity to access their level of understanding if different than other group members.
1. **Introduction**
   - Quick Set Induction to gain student interest.

2. **Group activity**
   - Students will get into pairs.
   - The pairs group will work together to construct various polygons either on a Geo Board or Cabri.
   - Using the constructions, the pairs will complete a table and answer various questions.
   - The teacher will monitor the groups to ensure all group members are participating and to assist with any questions

1. **Conclusion**
   - Class discussion of today’s discoveries
   - Class summary and journal assignment will conclude the lesson.

**Assessment:**
- Possible points for class and group participation will be 10. (This will take the place of a quiz score). Teacher will observe students to ensure that each student participates and assists other group members.
- Possible points for journal prompt will be 10 also.
- Each student will be given a review handout in two days that will also cover this material and it will be worth 30 points.
Napoleon Triangle

Activity One: Review of Macro

Team members’ names: ____________________________________________________

File name: ________________________________

Goal: To create a macro for equilateral triangles.

Using Cabri (Review of Macro’s)

1. Construct two points $A$ and $B$
2. Construct segment $\overline{AB}$
3. Construct a circle with center $A$ with radius $\overline{AB}$, and construct a circle with center $B$ with radius $\overline{AB}$
4. Label $C$ the intersection of the two circles (the intersection on the top)
5. Construct segments $\overline{AC}$ and $\overline{BC}$
6. Construct triangle $\triangle ABC$
7. Hide and show the two circles.

8. Now create a Macro, choose initial object, the two vertices $A$ and $B$
9. Choose the final object, the triangle.
10. Choose define Macro
11. Name your macro, and tell how to use it.

   (Remember: order counts and save the macro)
Napoleon Triangles
Activity Two

Team members’ names: __________________________________________________

Goal: To construct a napoleon triangle and to analyze some of its properties.

Now that you created your macro, let’s explore some of Napoleon’s proprieties.

Definition of Napoleon Triangle:
If we construct equilateral triangles on the sides of a triangle, the centers of those equilateral triangles themselves form an equilateral triangle.

1. Clear your screen.
2. Construct a triangle \( \triangle ABC \).
3. Select your macro
4. Choose two points, in the order you selected your macro. You should then have a triangle. Do this for the other segments.
5. Label the other three points \( D \), \( E \) and, \( F \).
6. Construct the segments \( AB \), \( AC \), \( BC \), \( BE \), \( CE \), \( CF \), \( AF \), \( AD \), and \( BD \).

7. Measure the angles and sides of the three triangles \( \triangle ABD \), \( \triangle ACF \), and \( \triangle BEC \).
8. What do you notice about the angles and the sides of the three triangles?
9. Find the centroid of each of these triangles and connect them with segments and the triangle, which is called the *outer Napoleon triangle*.

10. Label the points of the Napoleon triangle \(X\), \(Y\), and \(Z\).

11. Measure the angles and sides of this triangle.

12. What did you observe about the outer Napoleon triangle?

13. Connect angle \( \angle E \) to angle \( \angle A \), angle \( \angle D \) to angle \( \angle C \), and \( \angle F \) to angle \( \angle B \) 

14. Does this point have any special properties?

15. Move one of the vertices of the original triangle \( \triangle ABC \).

16. What happens to the sides and angles of the all of the triangles?

17. Pick a different vertex. What happens to the triangles?

18. Reflect each vertex of the Napoleon triangle with the closest side of the original triangle. \((X \text{ reflected with } \overline{AB}, Y \text{ reflected with } \overline{BC}, \text{ and } Z \text{ reflected with } \overline{AC})\)
19. Construct the triangle with those three points, which is called the *inner Napoleon triangle*.

![Inner Napoleon Triangle Diagram]

20. Measure the inner Napoleon triangle.

21. What did you notice about the triangle?

____________________________________________________________________

22. Measure the inner Napoleon triangle, the outer Napoleon triangle, and the original triangle.

23. What can you say about the three triangles?

____________________________________________________________________

24. Does the inner and outer Napoleon triangles have the same centers? Why or why not?

**Summarize in your own words the properties you have discovered.**