Global Instructions: (10 points) Solve each of the following problems without error. Show all details. Box in your answers. Use good notation, you will be marked off for bad notation.

(2 pts) 1. Let \( f \) be a differentiable function. Which of the following is equivalent to the definition of \( f'(a) \)? Select only one. Note: Read each alternative carefully.

\[
\begin{align*}
\Box f'(a) &= \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \\
\Box f'(a) &= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \\
\Box f'(a) &= \lim_{x \to 0} \frac{f(a) - f(x)}{a - x} \\
\Box f'(a) &= \lim_{h \to 0} \frac{f(a) - f(a + h)}{a - h} \\
\Box f'(a) &= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}
\end{align*}
\]

(4 pts) 2. Define the function \( f(x) = \frac{1}{\sqrt{x}} \). Compute \( f'(4) \) using the definition of derivative.

Solution: We separate the algebraic steps from the limit calculations, for clarity.

Calculations
From the side calculation, we have
\[
f'(4) = \lim_{h \to 0} \frac{f(4 + h) - f(4)}{h}
\]
\[
= \lim_{h \to 0} \frac{1}{2 \sqrt{4 + h}(2 + \sqrt{4 + h})}
\]
\[
= -\frac{1}{16}
\]

Side Calculations
\[
f(4 + h) - f(4) = \frac{1}{\sqrt{4 + h}} - \frac{1}{\sqrt{4}}
\]
\[
= \frac{1}{\sqrt{4 + h}} - \frac{1}{2}
\]
\[
= \frac{1}{2 \sqrt{4 + h}}(2 - \sqrt{4 + h})
\]
\[
= \frac{1}{2 \sqrt{4 + h}}(2 - \sqrt{4 + h}) \cdot \frac{2 + \sqrt{4 + h}}{2 + \sqrt{4 + h}}
\]
\[
= \frac{1}{2 \sqrt{4 + h}} \cdot \frac{4 - (4 + h)}{2 + \sqrt{4 + h}}
\]
\[
= \frac{1}{2 \sqrt{4 + h}(2 + \sqrt{4 + h})}
\]
and so...
\[
\frac{f(4 + h) - f(4)}{h} = -\frac{1}{2 \sqrt{4 + h}(2 + \sqrt{4 + h})}
\]

(4 pts) 3. Consider the function \( f(x) = 3x^3 + 4x^2 - 3x \), it is given that \( f'(-1) = -2 \). Find the equation of the line tangent to the graph of \( f \) at the point on the graph corresponding to \( x = -1 \) (or \( a = -1 \), if you prefer). Box in your final answer.

Solution: We use the point-slope form of the equation of a line, for which we need a point and a slope. The slope is \( m_{\text{tan}} = f'(-1) = -2 \), given. We need the point, this is the point of tangency. For \( x = -1 \), \( f(-1) = -3 + 4 + 3 = 4 \). We take \( (x_0, y_0) = (-1, 4) \), therefore. This is the point on the graph of \( f \) referenced by \( x = -1 \). The equation of the tangent line is then...

\[
y - y_0 = m(x - x_0) \quad \text{point-slope form}
\]
\[
y - 4 = -2(x + 1) \quad \text{plug in data}
\]
\[
y = -2x - 2 + 4 \quad \text{working towards slope-intercept form}
\]
\[
y = -2x + 2 \quad \text{slope-intercept form}
\]

Presenting the solution, the equation of the line tangent to the graph of \( f \) at \( x = -1 \) is

\[
y = -2x + 2
\]

That’s all folks.