Global Instructions: (10 points) Solve each of the following problems without error. Show all details. Box in your answers. Use good notation, you will be marked off for bad notation.

1. Evaluate the following integrals, possibly using the technique of substitution.

(a) (3 pts) \( \int \frac{x^3 \sin(x) + \sqrt{x}}{x^3} \, dx = \)

Solution: We separate the fraction and solve.

\[
\int \frac{x^3 \sin(x) + \sqrt{x}}{x^3} \, dx = \int \sin(x) + x^{-5/2} \, dx
\]

\[
= -\cos(x) - \frac{2}{3}x^{-3/2} + C
\]

(b) (3 pts) \( \int \frac{4}{(1 + 2x)^2} \, dx = \)

Solution: This is a Power Rule integral. Let \( u = 1 + 2x \), and so \( du = 2 \, dx \). We then have,

\[
\int \frac{4}{(1 + 2x)^2} \, dx = \frac{4}{2} \int (1 + 2x)^{-2} \, 2 \, dx
\]

\[
= 2 \int u^{-2} \, du
\]

\[
= -\frac{2}{u} + C
\]

\[
= -\frac{2}{1 + 2x} + C
\]

(c) (4 pts) \( \int_0^1 (3x + 1)^3 \, dx = \)

Solution: This is a Power Rule integral. Let \( u = 3x + 1 \), and \( du = 3 \, dx \).

\[
\int_0^1 (3x + 1)^3 \, dx = \frac{1}{3} \int_1^4 u^3 \, du
\]

\[
= \frac{1}{3} \left[ \frac{1}{4}u^4 \right]_1^4
\]

\[
= \frac{1}{3} \cdot \frac{1}{4} (4^4 - 1)
\]

\[
= \frac{255}{12}
\]

\[
= \frac{85}{4}
\]