1. Identify all numbers $x$ at which the function $f(x) = \frac{x + 2}{\sqrt{x - 1}}$ is continuous.

Solution: We require $x - 1 > 0$, or $x > 1$. In interval notation, the set of all numbers at which $f$ is continuous is $(1, \infty)$. □

2. Given $f(x) = \begin{cases} 3x^2 - 2x & x < -1 \\ 6x^2 + x & x \geq -1 \end{cases}$. Is this function (a) continuous at $x = -1$; (b) discontinuous with a removable discontinuity at $x = -1$; or (c) discontinuous with a jump discontinuity at $x = -1$? Justify your response.

Solution: Look at the left and right limits:

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 3x^2 - 2x = 5$$
$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 6x^2 + x = 5 = f(-1)$$

Thus, $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$. The two sided limit exists and $\lim_{x \to -1} f(x) = f(-1)$. This function is continuous at $x = -1$, the answer is (a). □

3. Define the function $f(x) = 3x^2 - 2x$. Use one of the the formulas

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad \text{or} \quad m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Then the slope of the line tangent to the graph of $f$ at the point (1, 1).

Solution:

Calculations

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} 3x + 1 \quad \text{from side calc} = 4$$

Side Calculations

$$f(x) - f(1) = 3x^2 - 2x - 1 = (x - 1)(3x + 1)$$

thus, the difference quotient is

$$\frac{f(x) - f(1)}{x - 1} = 3x + 1$$