THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Flash Cards

Trigonometric Equations

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WebTrig Flash Cards
AcroTEX eDucation Bundle

Find all values of t which solve the equation

$$\tan^2 t = 1.$$

Solve

$$\sin(2t) = \sin t$$

where $t \in [0, 2\pi]$.

Find all real numbers x for which

$$\sin^2 x = -\cos(2x).$$

Hint Soln
Next Home

Find the solution set for the equation

$$\cot t = \cos t$$

on the interval $[0, 2\pi]$.

Find all values of t for which

$$\sin^2 t = 2\sin t - 1.$$

Find all values of $\alpha \in [0°, 360°)$ for which

$$\cos(2\alpha) = 2\sin\alpha\cos\alpha.$$

Hint Soln
Next Home

Find all real solutions of the equation

$$\cos(\pi + t) = \sin t.$$

Solve

$$\sin\alpha\cos\alpha = 0$$

where $\alpha \in [0^\circ, 360^\circ)$.

Determine all $\alpha \in [0^\circ, 360^\circ)$ for which

$$\cos(2\alpha) = 2\sin^2\alpha.$$

Solve

$$1 + \tan^2 \alpha = \cos \alpha$$

where $\alpha \in [0°, 360°)$.

HINT

In order to find all values of t which solve the equation

$$\tan^2 t = 1$$

recall that the period of the tangent function is π so you may add πk where k is any integer to a solution to obtain another solution.

Hint Soln
Next Home

Answer:

$$\boxed{t = \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k \text{ for } k \in \mathbb{Z}}$$

Solution: To solve $\tan^2 t = 1$ first take the square root of both sides. This gives

$$\begin{aligned} & \tan^2 t = 1 & \\ \Longrightarrow \quad & \tan t = \pm 1 & \\ \Longrightarrow \quad & t = \frac{\pi}{4}, \frac{3\pi}{4} & t \in [0, \pi] \\ \Longrightarrow \quad & t = \frac{\pi}{4} + \pi k, \frac{3\pi}{4} + \pi k & \text{where } k \text{ is any integer.} \end{aligned}$$

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Hint Soln
Next Home

HINT

To solve

$$\sin(2t) = \sin t$$

for t in the interval $[0, \pi]$ use the double angle identity

$$\sin(2t) = 2 \sin t \cos t.$$

Answer: $\boxed{t = 0, \frac{\pi}{3}, \text{ or } \pi}$

Solution: To solve $\sin(2t) = \sin t$, subtract $\sin t$ from both sides of the equation then use the double angle identity for sine

$$\sin(2t) = 2\sin t\cos t.$$

This gives

$$\begin{aligned} & \sin(2t) - \sin t = 0 \\ \Longrightarrow \quad & 2\sin t\cos t - \sin t = 0 \\ \Longrightarrow \quad & \sin t\,(2\cos t - 1) = 0 \\ \Longrightarrow \quad & \sin t = 0, \cos t = \frac{1}{2}. \end{aligned}$$

The first equation $\sin t = 0$ implies $t = 0, \pi$ for t in the interval $[0, \pi]$ and the second equation $\cos t = \frac{1}{2}$ implies $t = \frac{\pi}{3}$ for t in the interval $[0, \pi]$ Combining these gives the answer $t = 0, \frac{\pi}{3}$, or π. ◀

HINT

To *determine all real numbers x* for which

$$\sin^2 x = -\cos(2x)$$

use the half angle identity

$$\sin^2 x = \frac{1 - \cos(2x)}{2}.$$

Answer: $\boxed{x = \frac{\pi}{2} + \pi k \text{ for } k \in \mathbb{Z}}$

Solution: To solve $\sin^2 x = -\cos(2x)$ first use the half angle identity

$$\sin^2 x = \frac{1 - \cos(2x)}{2}.$$

This gives

$$\begin{aligned} \frac{1-\cos(2x)}{2} &= -\cos(2x) \\ \Longrightarrow \quad 1 - \cos(2x) &= -2\cos(2x) \\ \Longrightarrow \quad 1 &= -\cos(2x) \\ \Longrightarrow \quad 2x &= \pi + 2\pi k \qquad \text{where } k \text{ is any integer.} \end{aligned}$$

This implies

$$x = \frac{\pi}{2} + \pi k \text{ where } k \text{ is any integer.}$$

HINT

To find the solution set for the equation

$$\cot t = \cos t$$

on the interval $[0, 2\pi]$ use the fact that

$$\cot t = \frac{\cos t}{\sin t}.$$

Answer: $\boxed{t = \frac{\pi}{2} + \pi k \text{ for any } k \in \mathbb{Z}}$

Solution: To solve $\cot t = \cos t$ first use the identity $\cot t = \frac{\cos t}{\sin t}$ to obtain

$$
\begin{aligned}
& & \frac{\cos t}{\sin t} &= \cos t \\
&\Longrightarrow & \frac{\cos t}{\sin t} - \cos t &= 0 \\
&\Longrightarrow & \cos t\left(\frac{1}{\sin t} - 1\right) &= 0 \\
&\Longrightarrow & \cos t\,(\csc t - 1) &= 0. \\
&\Longrightarrow & \cos t = 0 \text{ or } \csc t &= 1.
\end{aligned}
$$

The first equation $\cos t = 0$ gives $t = \frac{\pi}{2} + \pi k$ where k is any integer and the second equation $\csc t = 1$ gives $t = \frac{\pi}{2} + 2\pi k$ where k is any integer, so the solution set is $t = \left\{\frac{\pi}{2} + \pi k \text{ where } k \text{ is any integer}\right\}$. ◀

HINT

To find all values of t for which

$$\sin^2 t = 2\sin t - 1$$

factor the given expression.

Answer: $\boxed{t = \frac{\pi}{2} + 2\pi k \text{ for any } k \in \mathbb{Z}}$

Solution: To solve

$$\sin^2 t = 2\sin t - 1$$

first subtract $2\sin t - 1$ from both sides of the equation and factor to obtain

$$\begin{aligned} & \sin^2 t - 2\sin t + 1 = 0 \\ \Longrightarrow \quad & (\sin t - 1)^2 = 0 \\ \Longrightarrow \quad & \sin t - 1 = 0 \\ \Longrightarrow \quad & \sin t = 1. \end{aligned}$$

This equation gives $t = \frac{\pi}{2} + 2\pi k$ where k is any integer. ◀

HINT

In order to determine the values of

$$\alpha \in [0^\circ, 360^\circ)$$

for which

$$\cos(2\alpha) = 2\sin\alpha\cos\alpha$$

use the double angle formula for the sine function.

Answer: $\boxed{\alpha = 22.5^\circ, 112.\,5^\circ}$

Solution: To solve

$$\cos(2\alpha) = 2\sin\alpha\cos\alpha$$

first make a substitution using the identity

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha.$$

This gives

$$\begin{aligned} && \cos(2\alpha) &= \sin(2\alpha) \\ \Longrightarrow && \tan(2\alpha) &= 1 \\ \Longrightarrow && 2\alpha &= 45^\circ, 225^\circ \\ \Longrightarrow && \alpha &= 22.5^\circ, 112.5^\circ \qquad \text{for } \alpha \in [0^\circ, 360^\circ) \end{aligned}$$

HINT

In order to solve the equation

$$\cos(\pi + t) = \sin t$$

remember

$$\cos\pi = -1 \text{ and } \sin\pi = 0.$$

Answer: $\boxed{t = -\frac{\pi}{4} + \pi k \text{ for } k \in \mathbb{Z}}$

Solution: To solve $\cos(\pi + t) = \sin t$ first use the identity

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

to obtain

$$\begin{aligned} \cos\pi\cos t - \sin\pi\sin t &= \sin t \\ \Longrightarrow \qquad -\cos t &= \sin t \\ \Longrightarrow \qquad -1 &= \tan t \end{aligned}$$

The last equation implies that $t = -\frac{\pi}{4} + \pi k$ where k is any integer. ◀

HINT

To solve

$$\sin\alpha\cos\alpha = 0$$

for all $\alpha \in [0°, 360°)$ recall that two terms multiplied together to equal zero implies that at least one of the terms equals zero.

Answer: $\boxed{\alpha = 0°, 90°, 180°, 270°}$

Solution: To solve $\sin\alpha \cot\alpha = 0$ set each factor equal to zero and solve. This gives

$$\begin{aligned} & \sin\alpha = 0 \\ \Longrightarrow\quad & \alpha = 0°, 180° \qquad \text{or} \\ & \cos\alpha = 0 \\ \Longrightarrow\quad & \alpha = 90°, 270°. \end{aligned}$$

Therefore, when $\alpha \in [0°, 360°)$ the solutions are

$$\alpha = 0°, 90°, 180°, 270°.$$

HINT

To solve

$$\cos(2\alpha) = 2\sin^2\alpha$$

for $\alpha \in [0°, 360°)$ use the double angle identity for the cosine function

$$\cos(2\alpha) = 1 - 2\sin^2\alpha.$$

Answer: $\boxed{\alpha = 30°, 150°, 210°, 330°}$

Solution: To solve $\cos(2\alpha) = 2\sin^2\alpha$ first use the double angle identity for the cosine function

$$\cos(2\alpha) = 1 - 2\sin^2\alpha.$$

This gives

$$\begin{aligned} & 1 - 2\sin^2\alpha = 2\sin^2\alpha \\ \Longrightarrow \quad & 1 - 4\sin^2\alpha = 0 \\ \Longrightarrow \quad & (1 - 2\sin\alpha)(1 + 2\sin\alpha) = 0 \\ \Longrightarrow \quad & \sin\alpha = \pm\frac{1}{2}. \end{aligned}$$

Since $\alpha \in [0°, 360°)$, we have $\alpha = 30°, 150°, 210°, 330°$. ◀

HINT

To solve

$$1 + \tan^2 \alpha = \cos \alpha$$

where $\alpha \in [0°, 360°)$ use the identity

$$1 + \tan^2 \alpha = \sec^2 \alpha.$$

Answer: $\boxed{\alpha = 180^\circ}$

Solution: To solve

$$1 + \tan^2\alpha = \cos\alpha$$

first use the fundamental identity

$$1 + \tan^2\alpha = \sec^2\alpha.$$

This gives

$$\begin{aligned} & \sec^2\alpha = \cos\alpha \\ \Longrightarrow \quad & \sec^2\alpha = \frac{1}{\sec\alpha} \\ \Longrightarrow \quad & \sec^3\alpha = 1 \\ \Longrightarrow \quad & \sec\alpha = 1. \end{aligned}$$

The $\sec\alpha = 1$ will be true whenever $\cos\alpha = 1$ since $\sec\alpha = \frac{1}{\cos\alpha}$. For $\alpha \in [0^\circ, 360^\circ)$ this implies that $\alpha = 180^\circ$. ◀