THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Memory Cards

Trigonometric identities

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Begin

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Express $\sin t$ in terms of a cofunction of the angle t and name the type of identity.

Express $\cos t$ in terms of a cofunction of the angle t and name the type of identity.

State two reciprocal or quotient identities for $\tan t$.

Hint Soln Next

True or False:

$$\cot t = \frac{\sin t}{\cos t}.$$

If you answer False provide a correct identity.

Express $\sec t$ in terms of another trigonometric function of the angle t rad.

Hint Soln Next

True or False:

$$\csc t = \frac{1}{\sin t}.$$

If you answer False provide a correct identity.

Complete the following identities and name the type of identity:

$$\sin(-t) =$$

$$\sec(-t) = ?$$

Hint Soln Next

True or False:

$$\tan(-t) = -\tan t.$$

If you answer False provide a correct identity.

Hint Soln Next

Complete the expression

$$\cos(-t) = ?$$

and state type.

Hint Soln Next

Express $-\csc t$ in terms of the cosecant function.

Hint Soln Next

True or False:

$$\cot(-t) = -\cot t.$$

Hint Soln Next

Express the number 1 in terms of two trigonometric functions and state the type of identity.

Complete the expression

$$\tan^2 t + 1 = ?$$

and state its type.

Hint Soln Next

Express

$$\sec^2 t - 1$$

in terms of the tangent function.

Complete the expression

$$1 + \cot^2 t = ?$$

and state its type.

Express

$$\csc^2 t - 1$$

in terms of the cotangent function.

True or False: The ordered pair of real numbers $(\cos t, \sin t)$ represents a point on the unit circle for any radian measure t.

True or False:

$$\cos^2 t = (1 + \sin t)\,(1 - \sin t)\,.$$

Complete the expression

$$\cos(\alpha + \beta) = ?$$

and state the identity.

Complete the expression

$$\cos(\alpha + \beta) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\sin(\alpha + \beta) = ?$$

and state the identity.

Complete the expression

$$\sin(\alpha - \beta) = ?$$

and state the identity.

Complete the expression

$$\sin\alpha\cos\beta - \sin\beta\cos\alpha = ?$$

and state the identity.

Complete the expression

$$\sin\alpha\cos\beta + \sin\beta\cos\alpha = ?$$

and state the identity.

Complete the expression

$$\sin(\alpha + \beta) = ?$$

and state the identity.

Complete the expression

$$\cos\alpha\cos\beta - \sin\alpha\sin\beta = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\tan(\alpha + \beta) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\tan(\alpha - \beta) = ?$$

and state the identity.

Complete the expression

$$\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = ?$$

and state the identity.

Complete the expression

$$\sin\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\cos\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\tan\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\cot\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\sec\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\csc\left(\frac{\pi}{2} - t\right) = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\sin(2\alpha) = ?$$

and state the identity.

Complete the expression

$$\cos(2\alpha) = ?$$

and state the identity.

Complete the expression

$$\tan(2\alpha) = ?$$

and state the identity.

Complete the expression

$$\sin\frac{\alpha}{2} = ?$$

and state the identity.

Complete the expression

$$\cos\frac{\alpha}{2} = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\tan\left(\frac{\alpha}{2}\right)?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\cos\alpha\cos\beta = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\sin\alpha\sin\beta = ?$$

and state the identity.

Hint Soln Next

Complete the expression

$$\cos\alpha \sin\beta = ?$$

and state the identity.

True or False:

$$\cos\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right)$$

True or False:

$$\sin\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right)$$

True or False:

$$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$$

True or False:

$$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$$

True or False:

$$\cos\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)$$

Hint Soln Next

HINT

Let $P(x, y)$ be the point on the unit circle centered at $(0, 0)$ that determines the standard position angle of measure t rad. Then

$$\csc t = \frac{1}{y}$$

Soln Next

Answer: $\boxed{\sin t = \frac{1}{\csc t}}$

This identity is a **reciprocal** and **quotient** identity. ◀

HINT

Let $P(x, y)$ be the point on the unit circle centered at $(0, 0)$ that determines the standard position angle of measure t rad. Then

$$\sec t = \frac{1}{x}$$

Soln Next

$$\text{Answer: } \boxed{\cos t = \frac{1}{\sec t}}$$

This identity is a **reciprocal** and **quotient** identity. ◀

Next

HINT

The tangent function is related the sine and cosine function.

Answer: $\boxed{\tan t = \dfrac{1}{\cot t} = \dfrac{\sin t}{\cos t}}$

Next

HINT

Recall that
$$\tan t = \frac{\sin t}{\cos t}$$

Soln Next

Answer: $\boxed{\text{False:} \quad \cot t = \frac{\cos t}{\sin t}}$

HINT

The secant function is related to the cosine function.

$$\text{Answer: } \boxed{\sec t = \frac{1}{\cos t}}$$

This identity is a **reciprocal** and **quotient** identity. ◀

Next

HINT

This question has two answers. Recall that $\cot t = \dfrac{x}{y}$

Answer: $\boxed{\text{True}}$

Next

HINT

The sine function is odd. Recall that a function is odd if

$$f(-x) = -f(x)$$

for all x in the domain of f.

Soln Next

Answer:

$$\boxed{\sin(-t) = -\sin(t) \quad \text{and} \quad \sec(-t) = \sec t}$$

These are symmetric identities.

Next

HINT

The graph of the tangent function is given below.

Soln Next

Answer: $\boxed{True}$

This is a symmetric identity and it suggests that the tangent function is odd.

Next

HINT

Recall that the cosine function is even. A function f is even if

$$f(-x) = f(x)$$

for all x in the domain of f.

Soln Next

Answer: $\boxed{True}$

This is a symmetric identity. ◀

Next

HINT

The graph of the cosecant function is given below.

Soln Next

Answer: $\boxed{\csc(-t)}$

This is a symmetric identity.

Next

HINT

Recall the symmetric identities.

Soln Next

Answer: $\boxed{True}$

Next

HINT

Let (x, y) be the point on the unit circle with center $(0, 0)$ that determines the standard position angle t rad. Then

$$x^2 + y^2 = 1$$

Answer: $\boxed{\sin^2 t + \cos^2 t = 1}$

This is the first of the Pythagorean identities. ◀

HINT

Dividing through the identity

$$\sin^2 t + \cos^2 t = 1$$

by $\cos^2 t$ yields

$$\frac{\sin^2 t}{\cos^2 t} + \frac{\cos^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}.$$

Answer: $\boxed{\tan^2 t + 1 = \sec^2 t}$

This is a Pythagorean identity.

Next

HINT

Use a Pythagorean identity.

Soln Next

Answer: $\boxed{\sec^2 t - 1 = \tan^2 t}$

Next

HINT

Dividing through the identity

$$\sin^2 t + \cos^2 t = 1$$

by $\sin^2 t$ yields

$$\frac{\sin^2 t}{\sin^2 t} + \frac{\cos^2 t}{\sin^2 t} = \frac{1}{\sin^2 t}.$$

Soln Next

Answer: $\boxed{1 + \cot^2 t = \csc^2 t}$

This is a Pythagorean identity.

HINT

Use a Pythagorean identity.

Answer: $\boxed{\csc^2 t - 1 = \cot^2 t}$

Next

HINT

Recall that the equation of the unit circle is

$$x^2 + y^2 = 1.$$

Answer: $\boxed{True}$

The Pythagorean identity

$$\sin^2 t + \cos^2 = 1$$

verifies that $(\cos t, \sin t)$ lies on the unit circle since $x = \cos t$ and $y = \sin t$ satisfies

$$x^2 + y^2 = 1.$$

HINT

Recall the Pythagorean identities.

Soln Next

Answer: $\boxed{True}$

$$\cos^2 t = 1 - \sin^2 t = (1 + \sin t)(1 - \sin t)$$

Next

HINT

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - ?$$

Soln Next

Answer:

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

This is the sum formula for the cosine function.

HINT

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + ?$$

Answer:

$$\boxed{\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$$

This is the difference formula for the cosine function. ◀

HINT

$$\sin(\alpha + \beta) = ? + \sin\beta\cos\alpha$$

Soln Next

Answer:

$$\boxed{\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha}$$

This is the sum formula for the sine function.

HINT

$$\sin(\alpha - \beta) = ? - \sin\beta\cos\alpha$$

Soln Next

Answer:

$$\boxed{\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha}$$

This is the difference formula for the sine function.

HINT

Recall the difference formula for the sine function.

Soln Next

Answer:

$$\boxed{\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha}$$

This is the difference formula for the sine function.

Next

HINT

Recall the sum formula for the sine function.

Answer:

$$\boxed{\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha}$$

This is the difference formula for the sine function. ◀

HINT

$$\sin(\alpha+\beta) = ? + \sin\beta\cos\alpha$$

Soln Next

Answer:

$$\boxed{\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha}$$

This is the sum formula for the sine function. ◄

Next

HINT

Recall the sum formula for the cosine function.

Answer:

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

This is the sum formula for the cosine function.

Next

HINT

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{?}$$

Soln Next

Answer: $\boxed{\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}}$

This is the sum formula for the tangent function. ◀

HINT

Recall the sum formula for the tangent function.

Answer: $\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$

This is the sum formula for the tangent function. ◀

Next

HINT

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{?}$$

Answer: $\boxed{\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$

This is the difference formula for the tangent function. ◀

HINT

Recall the difference formula for the tangent function.

Soln Next

Answer: $\boxed{\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}}$

This is the sum formula for the tangent function. ◀

Next

HINT

If you have forgotten this you may construct the answer by applying the difference formula for the sine function.

Soln Next

Answer: $\boxed{\sin\left(\frac{\pi}{2} - t\right) = \cos t}$

The is a cofunction identity.

Next

HINT

If you have forgotten this you may construct the answer by applying the difference formula for the cosine function.

Soln Next

Answer: $\boxed{\cos\left(\frac{\pi}{2} - t\right) = \sin t}$

The is a cofunction identity.

Next

HINT

Observe that

$$\tan\left(\frac{\pi}{2} - t\right) = \frac{\sin\left(\frac{\pi}{2} - t\right)}{\cos\left(\frac{\pi}{2} - t\right)}.$$

Soln Next

Answer: $\boxed{\tan\left(\frac{\pi}{2} - t\right) = \cot t}$

The is a cofunction identity. ◀

Next

HINT

Observe that

$$\cot\left(\frac{\pi}{2}-t\right)=\frac{\cos\left(\frac{\pi}{2}-t\right)}{\sin\left(\frac{\pi}{2}-t\right)}.$$

Soln Next

Answer: $\boxed{\cot\left(\frac{\pi}{2} - t\right) = \tan t}$

The is a cofunction identity.

Next

HINT

Observe that

$$\sec\left(\frac{\pi}{2} - t\right) = \frac{1}{\cos\left(\frac{\pi}{2} - t\right)}.$$

Soln Next

$$\text{Answer: } \boxed{\sec\left(\frac{\pi}{2} - t\right) = \csc t}$$

The is a cofunction identity.

Next

HINT

Observe that

$$\csc\left(\frac{\pi}{2} - t\right) = \frac{1}{\sin\left(\frac{\pi}{2} - t\right)}.$$

Answer: $\boxed{\csc\left(\frac{\pi}{2} - t\right) = \sec t}$

The is a cofunction identity.

Next

HINT

If you have forgotten consider

$$\sin(\alpha + \alpha)$$

Soln Next

Answer: $\boxed{\sin(2\alpha) = 2\sin\alpha\cos\alpha}$

This is a double angle formula.

HINT

This question has three answers.

Answer: $$\boxed{\begin{aligned}\cos(2\alpha) &= \cos^2\alpha - \sin^2\alpha \\ &= 1 - 2\sin^2\alpha \\ &= 2\cos^2\alpha - 1\end{aligned}}$$

These are the three double angle formulas for the cosine function.

HINT

If you have forgotten consider

$$\tan(\alpha + \alpha)$$

Soln Next

Answer: $\boxed{\tan(2\alpha) = \frac{2\tan\alpha}{1-\tan^2\alpha}}$

This is a double angle formula. ◀

Next

HINT

If necessary you may used the double angle formula

$$\cos\alpha = \cos\left(2\frac{\alpha}{2}\right) = 1 - \sin^2\frac{\alpha}{2}$$

Answer: $\boxed{\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$

This is a half angle formula. ◀

Next

HINT

If necessary you may used the double angle formula

$$\cos\alpha = \cos\left(2\frac{\alpha}{2}\right) = 2\cos^2\frac{\alpha}{2} - 1$$

Soln Next

Answer: $\boxed{\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}}$

This is a half angle formula.

Next

HINT

This question has three answers.

Answer: $$\boxed{\begin{aligned} \tan\left(\tfrac{\alpha}{2}\right) &= \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} \\ &= \frac{\sin\alpha}{1+\cos\alpha} \\ &= \frac{1-\cos\alpha}{\sin\alpha} \end{aligned}}$$

This are the half angle formulas for the tangent function. ◀

HINT

Consider the sum

$$\cos(\alpha + \beta) + \cos(\alpha - \beta)$$

Soln Next

Answer:

$$\boxed{\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)}$$

This is a product formula. ◀

Next

HINT

Consider the sum

$$\cos(\alpha - \beta) - \cos(\alpha + \beta)$$

Soln Next

Answer:

$$\boxed{\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)}$$

This is a product formula. ◀

Next

HINT

Consider the sum

$$\sin(\alpha + \beta) - \sin(\alpha - \beta)$$

Soln Next

Answer:

$$\boxed{\cos\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right)}$$

This is a product formula.

Next

HINT

If you are not sure use the sum formulas on the expression

$$\cos\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right).$$

Answer: $\boxed{True}$

This is a product formula. ◀

Next

HINT

If you are not sure use the sum formulas on the

$$\frac{1}{2}\left(\sin(\alpha+\beta)-\sin(\alpha-\beta)\right).$$

Answer: $\boxed{False}$

Next

HINT

If you are not sure use the sum formulas on the expression

$$\frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right).$$

Soln Next

Answer: $\boxed{True}$

This is a product formula. $\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right)$

Next

HINT

If you are not sure use the sum formulas on the expression

$$\frac{1}{2}\left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right).$$

Soln Next

Answer: $\boxed{True}$

This is a product formula.

Next

HINT

If you are not sure use the sum formulas on the expression

$$\frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right).$$

Answer: $\boxed{False}$

Next