THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Flash Cards

Trigonometric Identities

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Verify that

$$\tan^2 + 1 = \sec^2 t.$$

Solve the equation

$$2\sin t + \cos^2 t = 2.$$

Show that
$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha.$$

Without a calculator determine the value of

$$\sin\frac{7\pi}{12}.$$

Hint Soln
Next Home

Without a calculator determine the value of

$$\cos 285°.$$

Without a calculator determine the value of

$$\tan 75°.$$

Without a calculator determine the value of

$$\tan \frac{5\pi}{12}.$$

Without a calculator determine the value of

$$\sin \frac{13\pi}{12}.$$

Verify the double-angle formula

$$\cos(2\alpha) = 2\cos^2\alpha - 1.$$

Verify the half-angle formula for the sine function:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}.$$

Without a calculator determine the value of

$$\sin 15^\circ.$$

Without a calculator determine the value of

$$\cos\frac{8\pi}{3}.$$

Hint Soln
Next Home

Verify the cofunction identity

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

for the value of $\sin\left(-\frac{\pi}{6}\right)$.

Hint Soln
Next Home

Verify the product formula

$$\cos\alpha\sin\beta$$

$$= \frac{1}{2}\left[\sin\left(\alpha+\beta\right) - \sin\left(\alpha-\beta\right)\right]$$

for the values

$$\alpha = \frac{\pi}{3} \text{ and } \beta = \frac{5\pi}{6}.$$

Hint Soln
Next Home

Verify the factor formula

$$\cos\alpha + \cos\beta$$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

for the values

$$\alpha = \frac{\pi}{3} \text{ and } \beta = \frac{2\pi}{3}.$$

Hint Soln
Next Home

Simplify the expression

$$\frac{\cos^2 t}{1 - \sin t}.$$

Show that

$$\sec t \cot t = \csc t.$$

Show that

$$\sin t \cot t = \cos t.$$

Verify that

$$\tan(-t) = -\tan t.$$

Show that

$$\cos 3a = 4\cos^3 a - 3\cos a.$$

HINT

To verify that

$$\tan^2 + 1 = \sec^2 t$$

use the Pythagorean identity involving sine and cosine.

Solution: Divide both sides of the Pythagorean identity

$$\sin^2 t + \cos^2 t = 1$$

by $\cos^2 t$ to obtain

$$\frac{\sin^2 t}{\cos^2 t} + 1 = \frac{1}{\cos^2 t}$$

$$\Longrightarrow \qquad \tan^2 + 1 = \sec^2 t.$$

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HINT

To solve the equation

$$2\sin t + \cos^2 t = 2$$

the pythagorean identity

$$\sin^2 t + \cos^2 t = 1.$$

Answer: $\boxed{t = \frac{\pi}{2} + 2\pi k}$

Solution: We know that $\sin^2 t + \cos^2 t = 1$, so $\cos^2 t = 1 - \sin^2 t$. Hence,

$$\begin{aligned} & 2\sin t + \cos^2 t = 2 \\ \Longrightarrow \quad & 2\sin t + 1 - \sin^2 t = 2 \\ \Longrightarrow \quad & 2\sin t - 1 - \sin^2 t = 0 \\ \Longrightarrow \quad & \sin^2 t - 2\sin t + 1 = 0 \\ \Longrightarrow \quad & (\sin t - 1)^2 = 0 \\ \Longrightarrow \quad & \sin t = 1 \end{aligned}$$

The sine function equals 1 whenever $t = \frac{\pi}{2} + 2\pi k$ for some integer k. ◀

HINT

To show that

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan\alpha$$

use the cofunction identity involving tangent.

Solution: Using the definition of the cotangent function and a similar cofunction identity for the tangent function, we have

$$\cot(\frac{\pi}{2} - \alpha) = \frac{1}{\tan(\frac{\pi}{2} - \alpha)} = \frac{1}{\cot \alpha} = \tan \alpha.$$

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HINT

To find $\sin \frac{7\pi}{12}$, notice that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$.

Answer: $\boxed{\sin\frac{7\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}}$

Solution: Notice $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, which are both basic angles we know from the unit circle. So,

$$\begin{aligned}
\sin\frac{7\pi}{12} &= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \\
&= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3} \\
&= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\
&= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{6}+\sqrt{2}}{4}.
\end{aligned}$$

HINT

To find

$$\cos 285°$$

use this the sum formula for the cosine function

$$\cos(a+b) = \cos a \cos b - \sin a \sin b.$$

Answer: $\boxed{\cos 285° = \frac{-\sqrt{2}-\sqrt{6}}{4}}$

Solution: Notice $285 = 240 + 45$. So,

$$\begin{aligned}\cos 285° &= \cos(240° + 45°)\\ &= \cos 240 \cos 45 - \sin 240 \sin 45\\ &= -\tfrac{1}{2} \cdot \tfrac{\sqrt{2}}{2} + \left(-\tfrac{\sqrt{3}}{2}\right) \cdot \tfrac{\sqrt{2}}{2}\\ &= \left(-\tfrac{\sqrt{2}}{4}\right) + \left(-\tfrac{\sqrt{6}}{4}\right)\\ &= \tfrac{-\sqrt{2}-\sqrt{6}}{4}.\end{aligned}$$

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HINT

To find

$$\tan 75^\circ$$

a the difference formula for tangent.

Answer: $\boxed{\tan 75^\circ = \dfrac{-1-\sqrt{3}}{1-\sqrt{3}}}$

Solution: Notice $75 = 135 - 60$. So,

$$\begin{aligned}\tan 75^\circ &= \tan\left(135^\circ - 60^\circ\right)\\ &= \frac{\tan 135 - \tan 60}{1 + \tan 135 \tan 60}\\ &= \frac{-1 - \sqrt{3}}{1 + (-1)\sqrt{3}}\\ &= \frac{-1 - \sqrt{3}}{1 - \sqrt{3}}.\end{aligned}$$

HINT

To find

$$\tan \frac{5\pi}{12}$$

find two numbers whose sum or difference is $\frac{5\pi}{12}$ and use the sum formula for tangent function.

Answer: $\boxed{\tan\frac{5\pi}{12} = \frac{1-\sqrt{3}}{\sqrt{3}-1}}$

Solution: Notice $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$. So,

$$\begin{aligned}\tan\frac{5\pi}{12} &= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)\\ &= \frac{\tan\frac{\pi}{6} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{6}\tan\frac{\pi}{4}}\\ &= \frac{\frac{1}{\sqrt{3}} - 1}{1 - \frac{1}{\sqrt{3}}\cdot 1}\\ &= \frac{1-\sqrt{3}}{\sqrt{3}-1}.\end{aligned}$$

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HINT

To find

$$\sin \frac{13\pi}{12}$$

use the difference formula for the sine function: $\sin(a - b) = \sin a \cos b - \sin b \cos a$.

Answer: $\boxed{\sin \frac{13\pi}{12} = \frac{-\sqrt{6}+\sqrt{2}}{4}}$

Solution: Notice $\frac{13\pi}{12} = \frac{4\pi}{3} - \frac{\pi}{4}$. So,

$$\begin{aligned} \sin \tfrac{13\pi}{12} &= \sin\left(\tfrac{4\pi}{3} - \tfrac{\pi}{4}\right) \\ &= \sin \tfrac{4\pi}{3} \cos \tfrac{\pi}{4} - \sin \tfrac{\pi}{4} \cos \tfrac{4\pi}{3} \\ &= \left(-\tfrac{\sqrt{3}}{2}\right) \tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\left(-\tfrac{1}{2}\right) \\ &= \tfrac{-\sqrt{6}}{4} + \tfrac{\sqrt{2}}{4} \\ &= \tfrac{-\sqrt{6}+\sqrt{2}}{4}. \end{aligned}$$

HINT

To verify the double-angle formula

$$\cos(2\alpha) = 2\cos^2\alpha - 1$$

use the fact that $2a = a + a$.

Solution: Since $2a = a + a$, use the cosine formula for the sum of two angles and a pythagorean identity to solve

$$\begin{aligned}\cos(2\alpha) &= \cos(\alpha + \alpha) \\ &= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha \\ &= \cos^2\alpha - \sin^2\alpha \\ &= \cos^2\alpha - (1 - \cos^2\alpha) \\ &= 2\cos^2\alpha - 1.\end{aligned}$$

HINT

To verify the half-angle formula for the sine function:

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}},$$

start with the double angle identity

$$\cos(2t) = 1 - 2\sin^2 t.$$

Solution: By the double angle formula for the cosine function we have

$$\cos(2t) = 1 - 2\sin^2 t$$
$$\Longrightarrow \qquad 2\sin^2 t = 1 - \cos(2t)$$

Dividing this last equation through by 2 and then taking the square root of both sides yields

$$\sin t = \pm\sqrt{\frac{1-\cos(2t)}{2}}.$$

Replacing t with $\frac{\alpha}{2}$ in this equation yields the desired half-angle formula

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}.$$

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HINT

To find $\sin 15°$, notice that

$$15 = \frac{30}{2}.$$

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Answer: $\boxed{\sin\frac{30}{2}^{\circ} = \frac{\sqrt{2-\sqrt{3}}}{2}}$

Solution: Notice $15 = \frac{30}{2}$, and 30° is a known basic angle. So, use the half angle formula for the sine function to solve.

$$= \sqrt{\frac{1-\cos 30^{\circ}}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}.$$

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HINT

To find $\cos \frac{8\pi}{3}$, notice that

$$\frac{8\pi}{3} = 2 \cdot \frac{4\pi}{3}$$

and use the double angle formula for cosine.

Hint Soln
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Answer: $\boxed{\cos\frac{8\pi}{3} = -\frac{1}{2}}$

Solution: Notice $\frac{8\pi}{3} = 2\cdot\frac{4\pi}{3}$ and use the double angle formula for cosine to obtain

$$\begin{aligned}\cos\left(2\cdot\tfrac{4\pi}{3}\right) &= 2\cos^2\tfrac{4\pi}{3} - 1\\ &= 2\left(-\frac{1}{2}\right)^2 - 1\\ &= 2\left(\frac{1}{4}\right) - 1\\ &= -\frac{1}{2}.\end{aligned}$$

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HINT

To verify the cofunction identity

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$

for the value of $\sin\left(-\frac{\pi}{6}\right)$, first set up an equation to find α.

Solution: First find α by solving

$$\begin{aligned} -\frac{\pi}{6} &= \frac{\pi}{2} - \alpha \\ \Longrightarrow \qquad \alpha &= \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}. \end{aligned}$$

Now, evaluate

$$\sin\left(-\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) = -\frac{1}{2} = \cos\frac{2\pi}{3}.$$

HINT

To verify the product formula

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

for the values

$$\alpha = \frac{\pi}{3} \text{ and } \beta = \frac{5\pi}{6}.$$

use

$$\sin(\alpha+\beta) = \sin\left(\tfrac{\pi}{3} + \tfrac{5\pi}{6}\right).$$

Answer: $\boxed{\text{Both yield the value } \frac{1}{4}}$

Solution: In this case,

$$\cos\alpha\sin\beta = \cos\tfrac{\pi}{3}\sin\tfrac{5\pi}{6} = \tfrac{1}{2}\cdot\tfrac{1}{2} = \tfrac{1}{4}.$$

Also,

$$\begin{aligned}\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right] &= \tfrac{1}{2}\left[\sin\left(\tfrac{\pi}{3}+\tfrac{5\pi}{6}\right)-\sin\left(\tfrac{\pi}{3}-\tfrac{5\pi}{6}\right)\right]\\ &= \tfrac{1}{2}\left[\sin\left(\tfrac{7\pi}{6}\right)-\sin\left(-\tfrac{\pi}{2}\right)\right]\\ &= \tfrac{1}{2}\left[-\tfrac{1}{2}-(-1)\right] = \tfrac{1}{4}.\end{aligned}$$

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HINT

To verify the factor formula

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

for the values $\alpha = \frac{\pi}{3}$ and $\beta = \frac{2\pi}{3}$, observe that $\frac{\alpha-\beta}{2} = -\frac{\pi}{6}$.

Answer: Both yield the value 0

Solution: In this case,

$$\cos\alpha + \cos\beta = \cos\frac{\pi}{3} + \cos\frac{2\pi}{3} = \frac{1}{2} + \left(-\frac{1}{2}\right) = 0.$$

Also,

$$\begin{aligned} 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) &= 2\cos\left(\frac{\frac{\pi}{3}+\frac{2\pi}{3}}{2}\right)\cos\left(\frac{\frac{\pi}{3}-\frac{2\pi}{3}}{2}\right) \\ &= 2\cos\left(\tfrac{\pi}{2}\right)\cos\left(\tfrac{-\pi}{6}\right) \\ &= 2\,(0)\left(\tfrac{\sqrt{3}}{2}\right) = 0. \end{aligned}$$

This means that both sides of the given equation are zero which establishes the desired equality. ◀

HINT

To simplify the expression

$$\frac{\cos^2 t}{1 - \sin t}$$

use the pythagorean identity

$$\cos^2 t = 1 - \sin^2 t.$$

Solution: Use the Pythagorean identities and factoring to establish

$$\begin{aligned} \frac{\cos^2 t}{1-\sin t} &= \frac{1-\sin^2 t}{1-\sin t} \\ \Longrightarrow \qquad &= \frac{(1-\sin t)(1+\sin t)}{1-\sin t} \\ \Longrightarrow \qquad &= 1+\sin t. \end{aligned}$$

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HINT

To show that

$$\sec t \cot t = \csc t$$

use the basic reciprocal identities

$$\sec t = \frac{1}{\cos t} \text{ and } \cot t = \frac{\cos t}{\sin t}.$$

Solution: Use the reciprocal identities to solve.

$$\begin{aligned}\sec t \cot t &= \frac{1}{\cos t} \cdot \frac{\cos t}{\sin t} \\ &= \frac{1}{\sin t} \\ &= \csc t.\end{aligned}$$

HINT

To show that

$$\sin t \cot t = \cos t$$

use the basic reciprocal identities.

Solution: Use the reciprocal identities to solve.

$$\begin{aligned} \sin t \cot t &= \sin t \cdot \frac{\cos t}{\sin t} \\ &= \cos t. \end{aligned}$$

HINT

To verify that

$$\tan(-t) = -\tan t,$$

remember that the sine function is odd and cosine is even. That is,

$$\sin(-t) = -\sin t \text{ and } \cos(-t) = \cos t.$$

Solution: Use the symmetry identities to solve, remembering that the sine function is odd and cosine is even.

$$\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\tan t.$$

HINT

To show that

$$\cos 3a = 4\cos^3 a - 3\cos a$$

use the fact that $3a = 2a + a$.

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Solution: Use the fact that $3a = 2a + a$ and the sum, difference, Pythagorean, and double angle formulas to arrive at the following:

$$\begin{aligned}
\cos 3a &= \cos(2a + a) \\
&= \cos 2a \cos a - \sin 2a \sin a \\
&= \left(2\cos^2 a - 1\right)\cos a - 2\sin a \cos a \sin a \\
&= 2\cos^3 a - \cos a - 2\cos a \sin^2 a \\
&= 2\cos^3 a - \cos a - 2\cos a\left(1 - \cos^2 a\right) \\
&= 2\cos^3 a - \cos a - 2\cos a + 2\cos^3 a \\
&= 4\cos^3 a - 3\cos a.
\end{aligned}$$

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