THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Flash Cards
Graphs of other Trigonometric Functions
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Graph the function

\[ y(t) = \tan \left( \frac{\pi}{2} t \right). \]
Graph the function

\[ y(t) = \tan(-2t + \pi). \]
Graph the function

\[ y(t) = \cot(2t) - 2. \]
Graph the function

\[ y(t) = \cot \left( \frac{\pi}{4} t + \frac{\pi}{4} \right). \]
Graph the function

\[ y(t) = \sec \left( \frac{\pi}{3} t \right) + 1. \]
Graph the function

\[ y(t) = \sec \left( \frac{1}{2} t - \frac{\pi}{2} \right). \]
Graph the function

\[ y(t) = \csc(-3t) + 2. \]
Graph the function

\[ y(t) = \csc\left(-t + \frac{\pi}{4}\right). \]
What is the period of the function

\[ y(t) = \tan(4t). \]
What is the period of the function

\[ y(t) = \cot \left( -\pi t + \frac{\pi}{4} \right). \]
What is the period of the function

\[ y(t) = \sec \left( \frac{-\pi}{3} t \right) - 1. \]
What is the period of the function

\[ y(t) = \csc \left( 3t - \frac{\pi}{3} \right) + 1. \]
Find a tangent function whose graph looks like the figure.
Find a cotangent function whose graph looks like the one given in the figure.
Find a secant function whose graph looks like the figure.
Find a cosecant function with a negative period whose graph looks like the given figure.
Find a tangent function whose graph is identical to the graph of the function

\[ y(t) = \cot[\pi(t + 1)] . \]
Find a cotangent function whose graph is identical to the graph of the function

\[ y(t) = \tan \left[ \frac{1}{2} (t + \pi) \right]. \]
Find a secant function whose graph is identical to the graph of the function

\[ y(t) = \csc \left[ 2 \left( t + \frac{\pi}{2} \right) \right]. \]
Find a cosecant function whose graph is identical to the graph of the function

$$y(t) = \sec \left[\frac{\pi}{2} (t + 2)\right] + 1.$$
HINT

In order to graph the function

\[ y(t) = \tan \left( \frac{\pi}{2} t \right) \]

note that the \( \frac{\pi}{2} \) causes a change in the graph’s period.
Solution: The graph of $y$ has the same general appearance as the fundamental tangent function except that the period is $\frac{\pi}{\frac{\pi}{2}} = 2$. 
HINT

In order to graph the function

\[ y(t) = \tan(-2t + \pi) \]

note that there are three changes in the appearance of the fundamental tangent function.
Solution: The graph of $y$ has the same general appearance as the fundamental tangent function except that the period is $\frac{\pi}{|-2|} = \frac{\pi}{2}$, there is a phase shift of $\frac{\pi}{2}$ units to the right and the graph is reflected over the $t$-axis since
\[
\tan (-2t + \pi) = \tan \left[ -2 \left( t - \frac{\pi}{2} \right) \right] \\
= -\tan \left[ 2 \left( t - \frac{\pi}{2} \right) \right].
\]
HINT

In order to graph the function

\[ y(t) = \cot(2t) - 2 \]

note that there is a change in the period, vertical shift, and reflection of the fundamental cotangent function.
Solution: The graph of $y$ has the same general appearance as the fundamental cotangent function except that the period is $\frac{\pi}{|-2|} = \frac{\pi}{2}$ and there is a shift of 2 units down.
HINT

In order to graph the function
\[ y(t) = \cot \left( \frac{\pi}{4} t + \frac{\pi}{4} \right) \]

note that there are two changes in the period and phase shift of the fundamental cotangent function.
Solution: The graph of $y$ has the same general appearance as the fundamental cotangent function except that the period is $\frac{\pi}{\frac{\pi}{4}} = 4$ and there is a phase shift of 1 unit to the left (since $\cot\left(\frac{\pi}{4}t + \frac{\pi}{4}\right) = \cot\left[\frac{\pi}{4}(t + 1)\right]$).
HINT

In order to graph the function

\[ y(t) = \sec\left(\frac{\pi}{3}t\right) + 1 \]

recall the fundamental secant function given in the following figure [figure].
Solution: The graph of $y$ has the same general appearance as the fundamental secant function except that the period is $\frac{2\pi}{\frac{\pi}{3}} = 6$ and there is a vertical shift of 1 unit up.
In order to graph the function

\[ y(t) = \sec \left( \frac{1}{2}t - \frac{\pi}{2} \right) \]

note that there are two changes in the appearance of the fundamental secant function.
Solution: The graph of \( y \) has the same general appearance as the fundamental secant function except that the period is \( \frac{2\pi}{\frac{1}{2}} = 4\pi \) and there is a phase shift of \( \pi \) units to the right since

\[
\sec \left( \frac{1}{2} t - \frac{\pi}{2} \right) = \sec \left[ \frac{1}{2} (t - \pi) \right].
\]
HINT

In order to graph the function

\[ y(t) = \csc(-3t) + 2 \]

recall the fundamental cosecant function given in the following figure.
Solution: The graph of $y$ has the same general appearance as the fundamental cosecant function except that the period is

$$\frac{2\pi}{|{-3}|} = \frac{2\pi}{3},$$

there is a shift of 2 units up, and the graph is reflected over the $t$-axis.
In order to graph the function

\[ y(t) = \csc \left( -t + \frac{\pi}{4} \right) \]

remember that \( \frac{\pi}{4} \) is a phase shift and the \(-t\) causes a reflection in the fundamental cosecant function.
Solution: The graph of $y$ has the same general appearance as the fundamental cosecant function except that there is a phase shift of $\frac{\pi}{4}$ units to the right (since $\csc(-t + \frac{\pi}{4}) = \csc[-(t - \frac{\pi}{4})]$) and the graph is reflected over the $t$-axis.
HINT

To find the period of the function

\[ y(t) = \tan(4t) \]

remember the fundamental tangent function has a period of \( \pi \).
Answer: $\frac{\pi}{4}$

Solution: The equation $y(t) = \tan(4t)$ has the form $y(t) = \tan(at)$ where the period is $\frac{\pi}{|a|}$. So, the period of $y$ is

$$\frac{\pi}{4} = \frac{\pi}{4}.$$
HINT

To find the period of the function

\[ y(t) = \cot \left( -\pi t + \frac{\pi}{4} \right) \]

put the function in the form

\[ y(t) = \cot (at + b) \]

then the period is

the period of \( \cot t \) \[ \frac{1}{|a|} \]
Answer: 1

Solution: The equation \( y(t) = \cot(-\pi t + \frac{\pi}{4}) \) has the form \( y(t) = \cot(at + b) \) where the period is \( \frac{\pi}{|a|} \). So, the period of \( y \) is

\[
\frac{\pi}{|\pi|} = 1.
\]

\( \Box \)
HINT

To find the period of the function

\[ y(t) = \sec\left(\frac{-\pi}{3}t\right) - 1 \]

remember the fundamental secant function has a period of \(2\pi\).
Answer: 6

Solution: The equation $y(t) = \sec \left( \frac{-\pi}{3} t \right) - 1$ has the form $y(t) = \sec(at + b) + c$ where the period is $\frac{2\pi}{|a|}$. So, the period of $y$ is

$$\frac{2\pi}{\left| \frac{-\pi}{3} \right|} = 6.$$
HINT

To find the period of the function

\[ y(t) = \csc \left( 3t - \frac{\pi}{3} \right) + 1 \]

note in this case the 3 is \( a \) in the function form \( y(t) = \csc (at + b) + c \).
Answer: \[ \frac{2\pi}{3} \]

Solution: The equation \( y(t) = \csc \left( 3t - \frac{\pi}{3} \right) + 1 \) has the form \( y(t) = \csc (at + b) + c \) where the period is \( \frac{2\pi}{|a|} \). So, the period of \( y \) is

\[ \frac{2\pi}{|3|} = \frac{2\pi}{3}. \]
HINT

To find a tangent function whose graph looks like the figure concentrate on one entire oscillation noting differences between the given graph and the fundamental tangent graph.
Answer: \[ y(t) = \tan\left(\frac{\pi}{2} t\right) \]

Solution: The function differs from the fundamental tangent wave in that the period is 2 units since the interval [0, 2] contains exactly one oscillation of \( y \). This means that \( \frac{\pi}{|a|} = 2 \) so that \( a = \frac{\pi}{2} \). This indicates that

\[ y(t) = \tan\left(\frac{\pi}{2} t\right). \]

There are many other acceptable answers for this question. Some include

\[ y_{a}(t) = \tan\left[\frac{\pi}{2} (t + 2)\right] \text{ and } \]
\[ y_{b}(t) = \tan\left[\frac{\pi}{2} (t - 2)\right]. \]

\[ \nabla \]
HINT

To find a cotangent function whose graph looks like the figure concentrate on the period and phase shift of the curve.
Answer: \[ y(t) = \cot \left( \frac{1}{2} t + \frac{\pi}{4} \right) \]

Solution: The function differs from the fundamental cotangent wave in that the period is \(2\pi\) units since the interval \([-\pi/2, 3\pi/2]\) contains exactly one oscillation of \(y\). This means that \(\frac{\pi}{|a|} = 2\pi\) so that \(a = \frac{1}{2}\). Also, the graph has been translated \(\frac{\pi}{2}\) units to the left. This indicates that

\[
\begin{align*}
y(t) &= \cot \left( \frac{1}{2} \left( t + \frac{\pi}{2} \right) \right) \\
&= \cot \left( \frac{1}{2} t + \frac{\pi}{4} \right). 
\end{align*}
\]

There are many other acceptable answers for this question. One includes

\[
y_a(t) = \cot \left( \frac{1}{2} \left( t - \frac{3\pi}{2} \right) \right).
\]
To find a secant function whose graph looks like the given figure consider the vertical asymptotes.
Answer: \[ y(t) = \sec[\pi t - \pi] + 1 \]

Solution: The function differs from the fundamental secant wave in that the period is 2 units since the interval \([0, 2]\) contains exactly one oscillation of \(y\). This means that \(\frac{2\pi}{|a|} = 2\) so that \(a = \pi\). Also, the graph has been translated 1 unit to the right and 1 unit up. This indicates that

\[
y(t) = \sec[\pi (t - 1)] + 1
= \sec[\pi t - \pi] + 1.
\]

There are many other acceptable answers for this question. Some include

\[
y_a(t) = -\sec(\pi t) + 1 \quad \text{and} \quad y_b(t) = \sec[\pi (t - 2)] + 1.
\]
HINT

To find a cosecant function whose graph looks like the figure note that only half of an entire oscillation of a cosecant wave appears in the figure.
Answer: \[ y(t) = \csc \left( \frac{2\pi}{3} t - \frac{\pi}{3} \right) \]

Solution: The function differs from the fundamental cosecant wave in that the period is 3 units since the interval \([0, 3]\) contains exactly one oscillation of \(y\). This means that \(\frac{2\pi}{|a|} = 3\) so we may choose \(a = \frac{2\pi}{3}\). Also, the graph has been translated \(\frac{1}{2}\) unit to the right. This indicates that

\[ y(t) = \csc \left[ \frac{2\pi}{3} \left( t - \frac{1}{2} \right) \right] = \csc \left[ \frac{2\pi}{3} t - \frac{\pi}{3} \right]. \]

There are many other acceptable answers for this question. Some include

\[ y_a(t) = -\csc \left[ \frac{2\pi}{3} \left( t + \frac{5}{2} \right) \right] \quad \text{and} \quad y_b(t) = -\csc \left[ \frac{2\pi}{3} \left( t - \frac{7}{2} \right) \right]. \]
HINT

To find a tangent function whose graph is identical to the graph of the function

$$y(t) = \cot[\pi(t + 1)]$$

use an appropriate phase shift.
Answer: \[ y_a (t) = - \tan \left[ \pi \left( t + \frac{3}{2} \right) \right] \]

Solution: The fundamental tangent function is the same basic shape as the fundamental cotangent function with a phase shift of \( \frac{\pi}{2} \) units (one half of the period) to the left and the graph is reflected over the \( t \)-axis. The period of the given function is \( \frac{\pi}{\pi} = 1 \) and one half of the period is \( \frac{1}{2} \). So, similarly reflect and shift the given function to get the answer.

\[ y_a (t) = - \tan \left[ \pi \left( t + \frac{3}{2} \right) \right] \]

There are many other acceptable answers for this question including, for example,

\[ y_b (t) = - \tan \left[ \pi \left( t + \frac{1}{2} \right) \right]. \]
HINT

To find a cotangent function whose graph is identical to the graph of the function

\[ y(t) = \tan \left( \frac{1}{2} (t + \pi) \right) \]

remember adding a negative sign in the appropriate place causes a reflection.
Answer: \[ y_a (t) = - \cot \left( \frac{1}{2} t \right) \]

Solution: The fundamental cotangent function is the same basic shape as the fundamental tangent function with a phase shift of \( \frac{\pi}{2} \) units (one half of the period) to the right and the graph is reflected over the \( t \)-axis. The period of the given function is \( \frac{\pi}{\frac{1}{2}} = 2\pi \) and one half of the period is \( \pi \). So, similarly reflect and shift the given function to get the answer.

\[ y_a (t) = - \cot \left( \frac{1}{2} t \right) \]

There are many other acceptable answers for this question. One includes

\[ y_b (t) = - \cot \left( \frac{1}{2} (t + \pi) \right). \]
To find a secant function whose graph is identical to the graph of the function \[ y(t) = \csc \left[ 2 \left( t + \frac{\pi}{2} \right) \right] \]
use an appropriate phase shift.
Answer: \[ y_a (t) = \sec \left[ 2 \left( t + \frac{3\pi}{4} \right) \right] \]

Solution: The fundamental cosecant function is the same basic shape as the fundamental secant function with a phase shift of \( \frac{\pi}{2} \) units (one forth of the period) to the left. The period of the given function is \( \frac{2\pi}{2} = \pi \) and one forth of the period is \( \frac{\pi}{4} \). So, similarly shift the given function to get the answer.

\[ y_a (t) = \sec \left[ 2 \left( t + \frac{3\pi}{4} \right) \right] \]

There are many other acceptable answers for this question. One includes

\[ y_b (t) = \sec \left[ 2 \left( t - \frac{\pi}{4} \right) \right] . \]
HINT

To find a cosecant function whose graph is identical to the graph of the function

\[ y(t) = \sec \left[ \frac{\pi}{2} (t + 2) \right] + 1 \]

compare the fundamental secant graph with the fundamental cosecant graph.
Answer: \( y_a (t) = \csc \left[ \frac{\pi}{2} (t - 1) \right] + 1 \)

Solution: The fundamental secant function is the same basic shape as the fundamental cosecant function with a phase shift of \( \frac{\pi}{2} \) units (one forth of the period) to the right. The period of the given function is \( \frac{2\pi}{\frac{\pi}{2}} = 4 \) and one forth of the period is 1. So, similarly shift the given function to get the answer.

\[
y_a (t) = \csc \left[ \frac{\pi}{2} (t - 1) \right] + 1
\]

There are many other acceptable answers for this question. One includes

\[
y_b (t) = - \csc \left[ \frac{\pi}{2} (t + 1) \right] + 1.
\]