

THE UNIVERSITY OF AKRON  
Theoretical and Applied Mathematics

*Flash Cards*

Graphs of other Trigonometric  
Functions

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and

Tom Price

**Instructions:** Click on the Begin button to view the first randomly selected card. Click on FS to view the cards in full screen mode (works only outside a web browser). The Home button on the first page goes to the WebTrig home page; otherwise, the Home button returns to this page. The Close button closes the document (use outside a web browser).





Graph the function

$$y(t) = \tan\left(\frac{\pi}{2}t\right).$$

Hint

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Graph the function

$$y(t) = \tan(-2t + \pi).$$

Hint

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Graph the function

$$y(t) = \cot(2t) - 2.$$

Hint

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Graph the function

$$y(t) = \cot\left(\frac{\pi}{4}t + \frac{\pi}{4}\right).$$

Hint

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Graph the function

$$y(t) = \sec\left(\frac{\pi}{3}t\right) + 1.$$

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Graph the function

$$y(t) = \sec\left(\frac{1}{2}t - \frac{\pi}{2}\right).$$

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Graph the function

$$y(t) = \csc(-3t) + 2.$$

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Graph the function

$$y(t) = \csc\left(-t + \frac{\pi}{4}\right).$$

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What is the period of the function

$$y(t) = \tan(4t).$$

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What is the period of the function

$$y(t) = \cot\left(-\pi t + \frac{\pi}{4}\right).$$

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What is the period of the function

$$y(t) = \sec\left(\frac{-\pi}{3}t\right) - 1.$$

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What is the period of the function

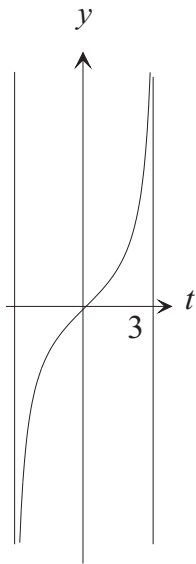
$$y(t) = \csc\left(3t - \frac{\pi}{3}\right) + 1.$$

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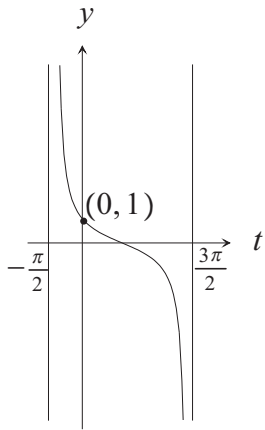
Find a tangent function whose graph looks like the figure.

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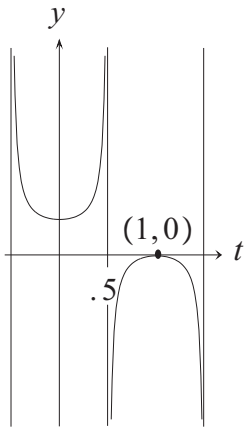
Find a cotangent function whose graph looks like the one given in the figure.

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Find a secant function whose graph looks like the figure.

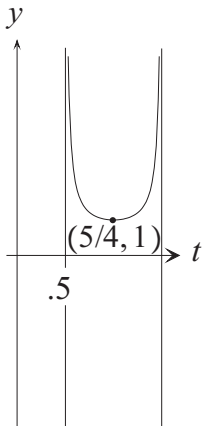
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Find a cosecant function with a negative period whose graph looks like the given figure..





Find a tangent function whose graph is identical to the graph of the function

$$y(t) = \cot [\pi (t + 1)].$$

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Find a cotangent function whose graph is identical to the graph of the function

$$y(t) = \tan \left[ \frac{1}{2} (t + \pi) \right].$$

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Find a secant function whose graph is identical to the graph of the function

$$y(t) = \csc \left[ 2 \left( t + \frac{\pi}{2} \right) \right].$$

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Find a cosecant function whose graph is identical to the graph of the function

$$y(t) = \sec \left[ \frac{\pi}{2} (t + 2) \right] + 1.$$

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## HINT

In order to graph the function

$$y(t) = \tan\left(\frac{\pi}{2}t\right)$$

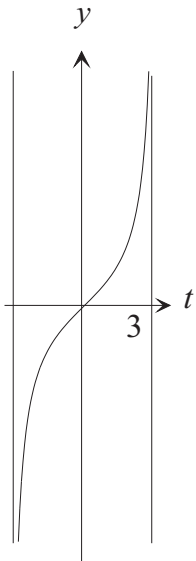
note that the  $\frac{\pi}{2}$  causes a change in the graph's period.

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Solution: The graph of  $y$  has the same general appearance as the fundamental tangent function except that the period is  $\frac{\pi}{|\frac{\pi}{2}|} = 2$ .





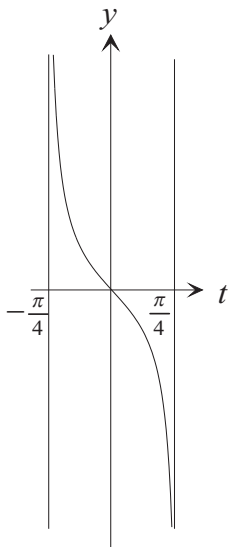
## HINT

In order to graph the function

$$y(t) = \tan(-2t + \pi)$$

note that there are three changes in the appearance of the fundamental tangent function.





Solution: The graph of  $y$  has the same general appearance as the fundamental tangent function except that the period is  $\frac{\pi}{|-2|} = \frac{\pi}{2}$ , there is a phase shift of  $\frac{\pi}{2}$  units to the right and the graph is reflected over the  $t$ -axis since

$$\begin{aligned}\tan(-2t + \pi) &= \tan\left[-2\left(t - \frac{\pi}{2}\right)\right] \\ &= -\tan\left[2\left(t - \frac{\pi}{2}\right)\right].\end{aligned}$$



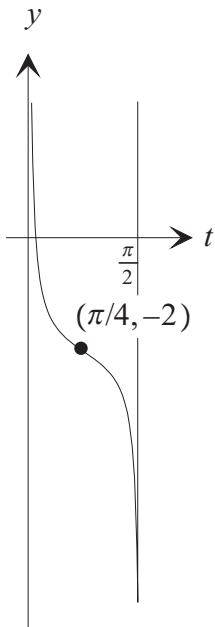


## HINT

In order to graph the function

$$y(t) = \cot(2t) - 2$$

note that there is a change in the period, vertical shift, and reflection of the fundamental cotangent function.



Solution: The graph of  $y$  has the same general appearance as the fundamental cotangent function except that the period is  $\frac{\pi}{|-2|} = \frac{\pi}{2}$  and there is a shift of 2 units down. ▶



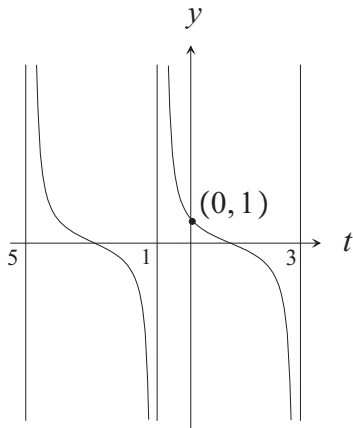


## HINT

In order to graph the function

$$y(t) = \cot\left(\frac{\pi}{4}t + \frac{\pi}{4}\right)$$

note that there are two changes in the period and phase shift of the fundamental cotangent function.



Solution: The graph of  $y$  has the same general appearance as the fundamental cotangent function except that the period is  $\frac{\pi}{|\frac{\pi}{4}|} = 4$  and there is a phase shift of 1 unit to the left (since  $\cot\left(\frac{\pi}{4}t + \frac{\pi}{4}\right) = \cot\left[\frac{\pi}{4}(t + 1)\right]$ ).



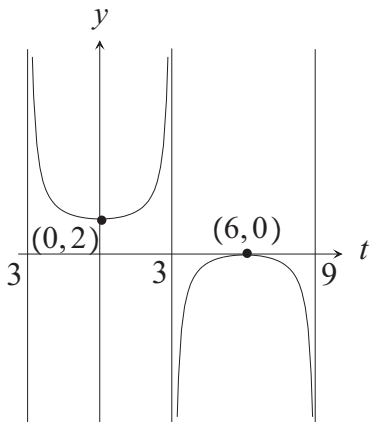


## HINT

In order to graph the function

$$y(t) = \sec\left(\frac{\pi}{3}t\right) + 1$$

recall the fundamental secant function given in the following figure [figure].



Solution: The graph of  $y$  has the same general appearance as the fundamental secant function except that the period is  $\frac{2\pi}{|\frac{\pi}{3}|} = 6$  and there is a vertical shift of 1 unit up.

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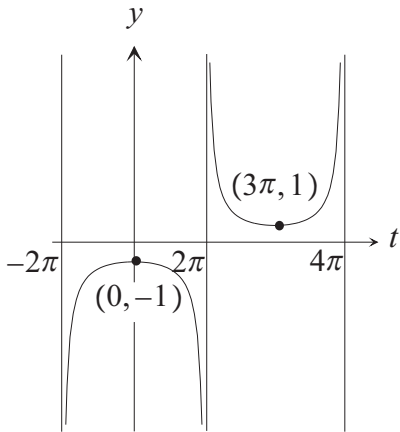
## HINT

In order to graph the function

$$y(t) = \sec\left(\frac{1}{2}t - \frac{\pi}{2}\right)$$

note that there are two changes in the appearance of the fundamental secant function.





Solution: The graph of  $y$  has the same general appearance as the fundamental secant function except that the period is  $\frac{2\pi}{|\frac{1}{2}|} = 4\pi$  and there is a phase shift of  $\pi$  units to the right since

$$\begin{aligned} & \sec\left(\frac{1}{2}t - \frac{\pi}{2}\right) \\ &= \sec\left[\frac{1}{2}(t - \pi)\right]. \end{aligned}$$



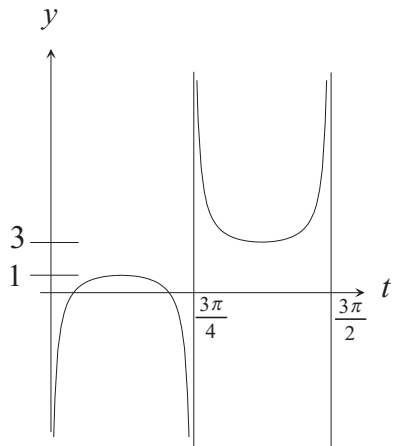


## HINT

In order to graph the function

$$y(t) = \csc(-3t) + 2$$

recall the fundamental cosecant function given in the following figure.



Solution: The graph of  $y$  has the same general appearance as the fundamental cosecant function except that the period is

$$\frac{2\pi}{|-3|} = \frac{2\pi}{3},$$

there is a shift of 2 units up, and the graph is reflected over the  $t$ -axis.



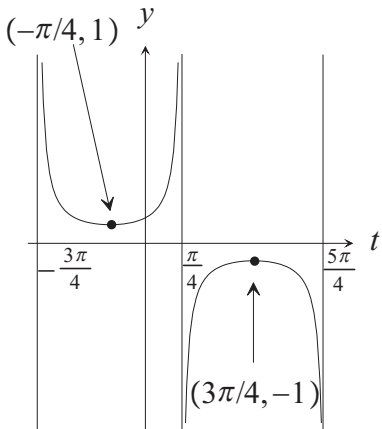


## HINT

In order to graph the function

$$y(t) = \csc\left(-t + \frac{\pi}{4}\right)$$

remember that  $\frac{\pi}{4}$  is a phase shift and the  $-t$  causes a reflection in the fundamental cosecant function.



Solution: The graph of  $y$  has the same general appearance as the fundamental cosecant function except that there is a phase shift of  $\frac{\pi}{4}$  units to the right (since  $\csc(-t + \frac{\pi}{4}) = \csc[-(t - \frac{\pi}{4})]$ ) and the graph is reflected over the  $t$ -axis.





## HINT

To find the period of the function

$$y(t) = \tan(4t)$$

remember the fundamental tangent function has a period of  $\pi$ .



Answer:  $\boxed{\frac{\pi}{4}}$

Solution: The equation  $y(t) = \tan(4t)$  has the form  $y(t) = \tan(at)$  where the period is  $\frac{\pi}{|a|}$ . So, the period of  $y$  is

$$\frac{\pi}{|4|} = \frac{\pi}{4}.$$





## HINT

To find the period of the function

$$y(t) = \cot\left(-\pi t + \frac{\pi}{4}\right)$$

put the function in the form  
 $y(t) = \cot(at + b)$  then the period is  
 $\frac{\text{the period of } \cot t}{|a|}$ .





Answer:  $\boxed{1}$

Solution: The equation  $y(t) = \cot\left(-\pi t + \frac{\pi}{4}\right)$  has the form  $y(t) = \cot(at + b)$  where the period is  $\frac{\pi}{|a|}$ . So, the period of  $y$  is

$$\frac{\pi}{|-\pi|} = 1.$$





## HINT

To find the period of the function

$$y(t) = \sec\left(\frac{-\pi}{3}t\right) - 1$$

remember the fundamental secant function has a period of  $2\pi$ .



Answer:  $\boxed{6}$

Solution: The equation  $y(t) = \sec\left(\frac{-\pi}{3}t\right) - 1$  has the form  $y(t) = \sec(at + b) + c$  where the period is  $\frac{2\pi}{|a|}$ . So, the period of  $y$  is

$$\frac{2\pi}{\left|\frac{-\pi}{3}\right|} = 6.$$





## HINT

To find the period of the function

$$y(t) = \csc\left(3t - \frac{\pi}{3}\right) + 1$$

note in this case the 3 is  $a$  in the function form  $y(t) = \csc(at + b) + c$ .

Answer:  $\boxed{\frac{2\pi}{3}}$

Solution: The equation  $y(t) = \csc(3t - \frac{\pi}{3}) + 1$  has the form  $y(t) = \csc(at + b) + c$  where the period is  $\frac{2\pi}{|a|}$ . So, the period of  $y$  is

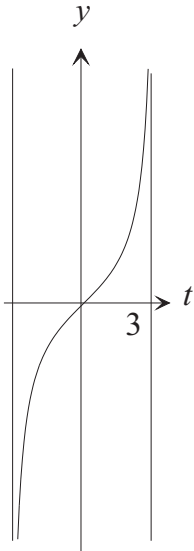
$$\frac{2\pi}{|3|} = \frac{2\pi}{3}.$$





## HINT

To find a tangent function whose graph looks like the figure concentrate on one entire oscillation noting differences between the given graph and the fundamental tangent graph.



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Answer:  $y(t) = \tan\left(\frac{\pi}{2}t\right)$

Solution: The function differs from the fundamental tangent wave in that the period is 2 units since the interval  $[0, 2]$  contains exactly one oscillation of  $y$ . This means that  $\frac{\pi}{|a|} = 2$  so that  $a = \frac{\pi}{2}$ . This indicates that

$$y(t) = \tan\left(\frac{\pi}{2}t\right).$$

There are many other acceptable answers for this question. Some include

$$y_a(t) = \tan\left[\frac{\pi}{2}(t+2)\right] \text{ and}$$

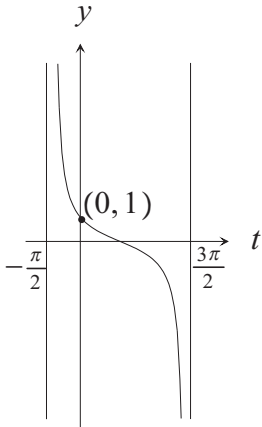
$$y_b(t) = \tan\left[\frac{\pi}{2}(t-2)\right].$$





## HINT

To find a cotangent function whose graph looks like the figure concentrate on the period and phase shift of the curve.



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$$\text{Answer: } \boxed{y(t) = \cot \left[ \frac{1}{2}t + \frac{\pi}{4} \right]}$$

Solution: The function differs from the fundamental cotangent wave in that the period is  $2\pi$  units since the interval  $[-\pi/2, 3\pi/2]$  contains exactly one oscillation of  $y$ . This means that  $\frac{\pi}{|a|} = 2\pi$  so that  $a = \frac{1}{2}$ . Also, the graph has been translated  $\frac{\pi}{2}$  units to the left. This indicates that

$$\begin{aligned} y(t) &= \cot \left[ \frac{1}{2} \left( t + \frac{\pi}{2} \right) \right] \\ &= \cot \left[ \frac{1}{2}t + \frac{\pi}{4} \right]. \end{aligned}$$

There are many other acceptable answers for this question. One includes

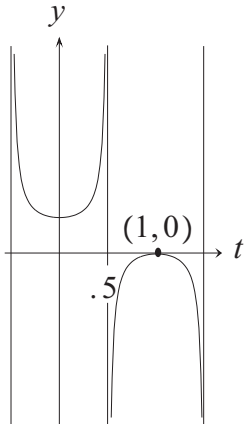
$$y_a(t) = \cot \left[ \frac{1}{2} \left( t - \frac{3\pi}{2} \right) \right].$$





## HINT

To find a secant function whose graph looks like the given figure consider the vertical asymptotes..



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Answer:  $y(t) = \sec[\pi t - \pi] + 1$

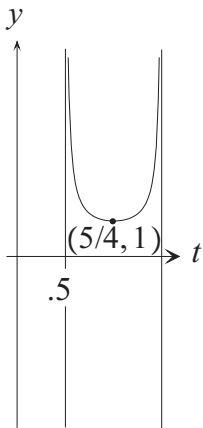
Solution: The function differs from the fundamental secant wave in that the period is 2 units since the interval  $[0, 2]$  contains exactly one oscillation of  $y$ . This means that  $\frac{2\pi}{|a|} = 2$  so that  $a = \pi$ . Also, the graph has been translated 1 unit to the right and 1 unit up. This indicates that

$$\begin{aligned}y(t) &= \sec[\pi(t - 1)] + 1 \\ &= \sec[\pi t - \pi] + 1.\end{aligned}$$

There are many other acceptable answers for this question. Some include

$$\begin{aligned}y_a(t) &= -\sec(\pi t) + 1 \text{ and} \\ y_b(t) &= \sec[\pi(t - 2)] + 1.\end{aligned}$$





## HINT

To find a cosecant function whose graph looks like the figure note that only half of an entire oscillation of a cosecant wave appears in the figure.

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Answer:  $y(t) = \csc\left(\frac{2\pi}{3}t - \frac{\pi}{3}\right)$

Solution: The function differs from the fundamental cosecant wave in that the period is 3 units since the interval  $[0, 3]$  contains exactly one oscillation of  $y$ . This means that  $\frac{2\pi}{|a|} = 3$  so we may choose  $a = \frac{2\pi}{3}$ . Also, the graph has been translated  $\frac{1}{2}$  unit to the right. This indicates that

$$y(t) = \csc\left[\frac{2\pi}{3}\left(t - \frac{1}{2}\right)\right] = \csc\left[\frac{2\pi}{3}t - \frac{\pi}{3}\right].$$

There are many other acceptable answers for this question. Some include

$$y_a(t) = -\csc\left[\frac{2\pi}{3}\left(t + \frac{5}{2}\right)\right] \text{ and } y_b(t) = -\csc\left[\frac{2\pi}{3}\left(t - \frac{7}{2}\right)\right].$$





## HINT

To find a tangent function whose graph is identical to the graph of the function

$$y(t) = \cot[\pi(t + 1)]$$

use an appropriate phase shift.



Answer: 
$$y_a(t) = -\tan\left[\pi\left(t + \frac{3}{2}\right)\right]$$

Solution: The fundamental tangent function is the same basic shape as the fundamental cotangent function with a phase shift of  $\frac{\pi}{2}$  units (one half of the period) to the left and the graph is reflected over the  $t$ -axis. The period of the given function is  $\frac{\pi}{\pi} = 1$  and one half of the period is  $\frac{1}{2}$ . So, similarly reflect and shift the given function to get the answer.

$$y_a(t) = -\tan\left[\pi\left(t + \frac{3}{2}\right)\right]$$

There are many other acceptable answers for this question including, for example,

$$y_b(t) = -\tan\left[\pi\left(t + \frac{1}{2}\right)\right].$$



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## HINT

To find a cotangent function whose graph is identical to the graph of the function

$$y(t) = \tan \left[ \frac{1}{2} (t + \pi) \right]$$

remember adding a negative sign in the appropriate place causes a reflection.





Answer:  $y_a(t) = -\cot\left(\frac{1}{2}t\right)$

Solution: The fundamental cotangent function is the same basic shape as the fundamental tangent function with a phase shift of  $\frac{\pi}{2}$  units (one half of the period) to the right and the graph is reflected over the  $t$ -axis. The period of the given function is  $\frac{\pi}{\frac{1}{2}} = 2\pi$  and one half of the period is  $\pi$ . So, similarly reflect and shift the given function to get the answer.

$$y_a(t) = -\cot\left(\frac{1}{2}t\right)$$

There are many other acceptable answers for this question. One includes

$$y_b(t) = -\cot\left[\frac{1}{2}(t + \pi)\right].$$





## HINT

To find a secant function whose graph is identical to the graph of the function

$$y(t) = \csc \left[ 2 \left( t + \frac{\pi}{2} \right) \right]$$

use an appropriate phase shift.



Answer: 
$$y_a(t) = \sec \left[ 2 \left( t + \frac{3\pi}{4} \right) \right]$$

Solution: The fundamental cosecant function is the same basic shape as the fundamental secant function with a phase shift of  $\frac{\pi}{2}$  units (one fourth of the period) to the left. The period of the given function is  $\frac{2\pi}{2} = \pi$  and one fourth of the period is  $\frac{\pi}{4}$ . So, similarly shift the given function to get the answer.

$$y_a(t) = \sec \left[ 2 \left( t + \frac{3\pi}{4} \right) \right]$$

There are many other acceptable answers for this question. One includes

$$y_b(t) = \sec \left[ 2 \left( t - \frac{\pi}{4} \right) \right].$$



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## HINT

To find a cosecant function whose graph is identical to the graph of the function

$$y(t) = \sec \left[ \frac{\pi}{2} (t + 2) \right] + 1$$

compare the fundamental secant graph with the fundamental cosecant graph.



Answer: 
$$y_a(t) = \csc \left[ \frac{\pi}{2} (t - 1) \right] + 1$$

Solution: The fundamental secant function is the same basic shape as the fundamental cosecant function with a phase shift of  $\frac{\pi}{2}$  units (one fourth of the period) to the right. The period of the given function is  $\frac{2\pi}{|\frac{\pi}{2}|} = 4$  and one fourth of the period is 1. So, similarly shift the given function to get the answer.

$$y_a(t) = \csc \left[ \frac{\pi}{2} (t - 1) \right] + 1$$

There are many other acceptable answers for this question. One includes

$$y_b(t) = -\csc \left[ \frac{\pi}{2} (t + 1) \right] + 1.$$

