THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Flash Cards

Graphs of the sine and cosine functions

Katie Jones
and
Tom Price

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State the amplitude of the function

$$y\,(t) = -4\sin\left[\frac{\pi}{3}\,(t+\pi)\right].$$

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State the amplitude of the function

$$y(t) = \frac{1}{5}\cos\left[3\left(t + \frac{\pi}{5}\right)\right].$$

Hint Soln
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State the period of the function

$$y(t) = 3\cos(2t+6)+7.$$

State the period of the function

$$y(t) = \frac{2}{3} \sin\left(-\frac{1}{3}t\right) + \frac{4}{3}.$$

For the following function find the phase shift and its direction

$$y(t) = -\sin(t + 3\pi) + 1.$$

For the following function find the phase shift and its direction

$$y\left(t\right) = \cos\left(\frac{2\pi}{3}t - \pi\right).$$

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Graph the function

$$y(t) = 3\sin t.$$

Hint Soln
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Graph the function

$$y(t) = -\sin(2t) + \frac{1}{2}.$$

Hint Soln
Next Home

Graph the function

$$y(t) = \frac{1}{4}\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}\right).$$

Graph the function

$$y\left(t\right) = \sin\left(t - \frac{\pi}{4}\right) - 2.$$

Graph the function

$$y\left(t\right)=4\cos\left(-\frac{1}{2}t\right).$$

Graph the function

$$y(t) = -\frac{3}{4}\cos(2t + 4).$$

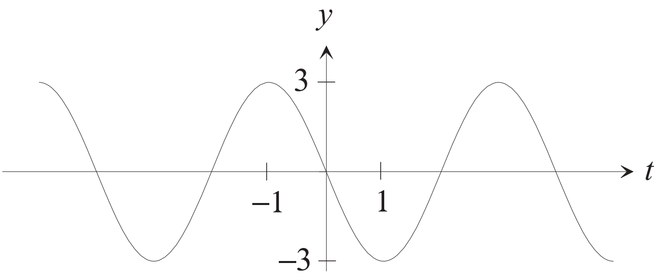
Graph the function

$$y(t) = \cos(6t) - 3.$$

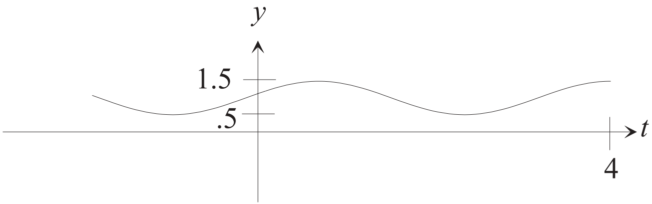
Graph the function

$$y\left(t\right) = \frac{1}{4}\cos\left[\frac{\pi}{4}\left(t-1\right)\right].$$

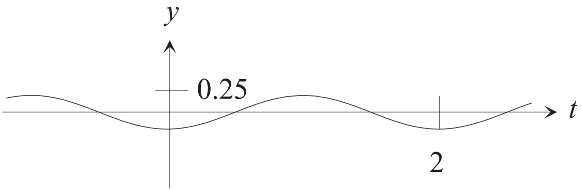
Find a sine function whose graph looks like the following.



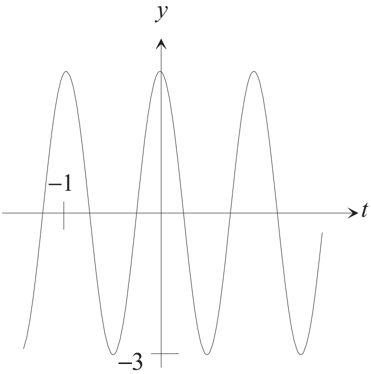
Find a sine function whose graph looks like the following



Find a cosine function whose graph is the following.



Find a cosine function that graphs



Find a cosine function whose graph is identical to the graph of the function

$$y(t) = 5\sin\left(\frac{1}{2}t + \frac{\pi}{2}\right).$$

Hint Soln
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Find a sine function whose graph is identical to the graph of the function

$$y(t) = \frac{1}{2}\cos\left[\pi(t+1)\right] + 1.$$

HINT

To find the amplitude of the function

$$y(t) = -4\sin\left[\frac{\pi}{3}(t+\pi)\right]$$

remember it is defined as the maximum vertical deviation of the function's graph from the t-axis.

Answer: $\boxed{4}$

Solution: The function $y(t) = -4\sin\left(\frac{\pi}{3}t + \pi\right)$ is in the form $f(t) = A\sin[a(t+b)]$. By definition, the amplitude of $f(t)$ is $|A|$, so for $y(t)$, the amplitude is

$$|-4| = 4.$$

HINT

To find the amplitude of the function

$$y(t) = \frac{1}{5}\cos\left[3\left(t + \frac{\pi}{5}\right)\right]$$

recall this is defined as the maximum vertical deviation of the function's graph from the t-axis.

Answer: $\boxed{\dfrac{1}{5}}$

Solution: The function, $y(t) = \frac{1}{5}\cos\left(3t + \frac{\pi}{5}\right)$ is in the form $f(t) = A\cos[a(t+b)]$. By definition, the amplitude of $f(t)$ is $|A|$, so for $y(t)$, the amplitude is

$$\left|\frac{1}{5}\right| = \frac{1}{5}.$$

HINT

To find the period of the function

$$y\,(t) = 3\cos\,(2t + 6) + 7$$

recall it is defined as the distance measured on the t-axis that it takes for the graph to complete one oscillation.

Answer: $\boxed{\pi}$

Solution: The period of $f(t) = A\cos[a(t+b)] + c$ is defined to be $\frac{2\pi}{|a|}$. Writing

$$y(t) = 3\cos(2t+6) + 7 = 3\cos[2(t+3)] + 7$$

we see that $a = 2$ so y has period

$$\frac{2\pi}{|2|} = \pi.$$

HINT

To find the period of the function

$$y(t) = \frac{2}{3}\sin\left(-\frac{1}{3}t\right) + \frac{4}{3}$$

recall that the period of

$$f(t) = A\sin\left[a\left(t+b\right)\right] + c$$

is $\frac{2\pi}{|a|}$.

Answer: $\boxed{6\pi}$

Solution: The function, $y(t) = \frac{2}{3}\sin\left(-\frac{1}{3}t\right) + \frac{4}{3}$ is in the form $f(t) = A\sin[a(t+b)] + c$. The period of $f(t)$ is defined to be $\frac{2\pi}{|a|}$, so for $y(t)$, the period is

$$\frac{2\pi}{\left|-\frac{1}{3}\right|} = 6\pi.$$

HINT

To find the phase shift including its direction for the function

$$y(t) = -\sin(t + 3\pi) + 1$$

remember the pahse shift is defined as the distance a graph is translated along the t-axis (right or left).

Answer: $\boxed{3\pi \text{ units to the left}}$

Solution: The function, $y(t) = -\sin(t + 3\pi) + 1$ is in the form $f(t) = A\sin[a(t + b)] + c$. The phase shift for $f(t)$ is b, so for $y(t)$, the phase shift is

$$3\pi.$$

This moves the graph to the left since the phase shift is positive.

◄

HINT

To find the phase shift including its direction for the function

$$y(t) = \cos\left(\frac{2\pi}{3}t - \pi\right)$$

put the function into the form $f(t) = A\cos[a(t+b)] + c$ and recall that the phase shift is determined by b.

Answer: $\boxed{-\frac{3}{2} \text{ or } \frac{3}{2} \text{ units to the right}}$

Solution: The function, $y(t) = \cos\left(\frac{2\pi}{3}t - \pi\right)$ needs to be in the form $f(t) = A\cos[a(t+b)] + c$. So write,

$$\cos\left(\frac{2\pi}{3}t - \pi\right) = \cos\left[\frac{2\pi}{3}\left(t - \frac{3}{2}\right)\right].$$

The phase shift for $f(t)$ is defined to be b, so for $y(t)$, the phase shift is

$$-\frac{3}{2}.$$

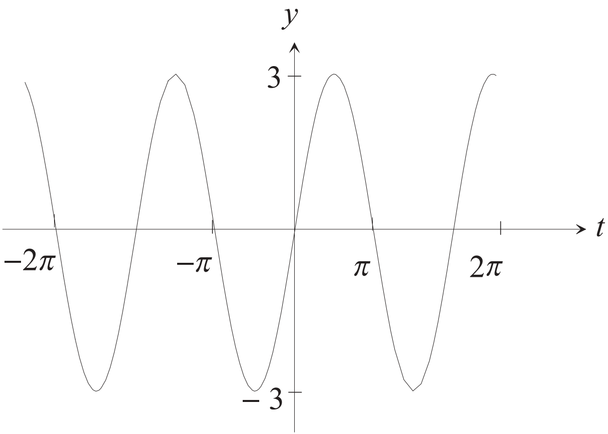
This moves the graph to the right since the phase shift is negative. ◀

HINT

To graph the function

$$y(t) = 3 \sin t$$

notice that the 3 causes a change in the graph's amplitude.



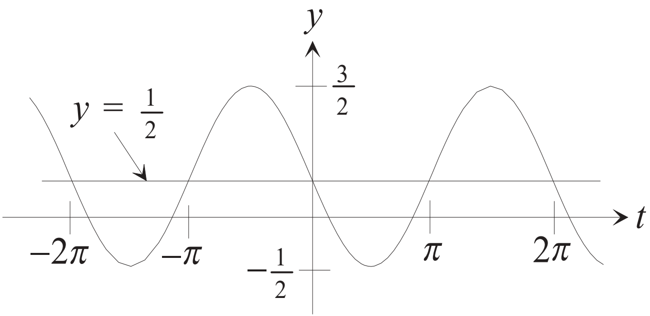
Solution: The graph of $y = 3\sin t$ has the same general appearance as the fundamental sine function except that the amplitude is 3 units. ◄

HINT

To graph the function

$$y(t) = -\sin(2t) + \frac{1}{2}$$

notice that there are three changes to the fundamental sine graph, one being that the negative sign causes a reflection over the t-axis.



Solution: The graph of $y = -\sin(2t) + \frac{1}{2}$ has the same general appearance as the fundamental sine function except that the period is $\frac{2\pi}{|2|} = \pi$, the entire graph is shifted up $\frac{1}{2}$ unit, and the graph is reflected over the line $y = \frac{1}{2}$. ◀

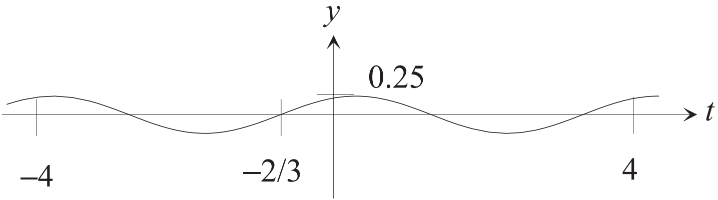
HINT

To graph the function

$$y(t) = \frac{1}{4}\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}\right)$$

put it in the form

$$f(t) = A\sin\left[a(t+b)\right] + c.$$



Solution: The function, $y(t) = \frac{1}{4}\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}\right)$ needs to be in the form $f(t) = A\sin[a(t+b)] + c$. So write,

$$y(t) = \frac{1}{4}\sin\left(\frac{\pi}{2}t + \frac{\pi}{3}\right) = \frac{1}{4}\sin\left[\frac{\pi}{2}\left(t + \frac{2}{3}\right)\right]$$

Then, y has a period of $\dfrac{2\pi}{\left|\frac{\pi}{2}\right|} = 4$ and an amplitude of $\frac{1}{4}$. Also, y has a phase shift of $\dfrac{2}{3}$ units to the left. ◀

HINT

To graph the function

$$y(t) = \sin\left(t - \frac{\pi}{4}\right) - 2$$

notice there are two changes in the fundamental sine graph.

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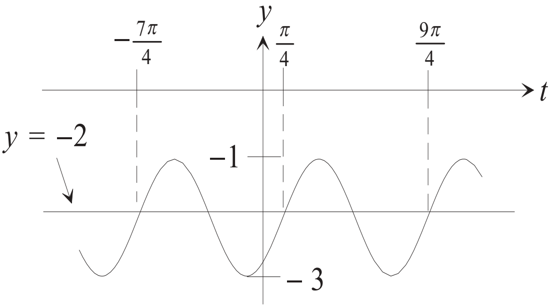


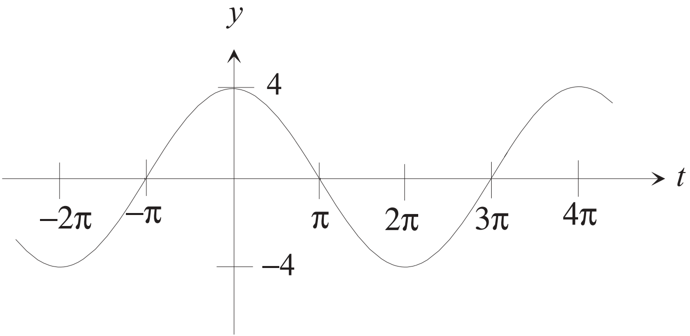
Solution: The graph of $y = \sin\left(t - \frac{\pi}{4}\right) - 2$ has the same general appearance as the fundamental sine function except that there is a phase shift of $\frac{\pi}{4}$ units to the right and a vertical shift of 2 units down. ◀

HINT

To graph the function

$$y(t) = 4\cos\left(-\frac{1}{2}t\right)$$

notice there is a change in the period and the amplitude of the fundamental cosine graph.



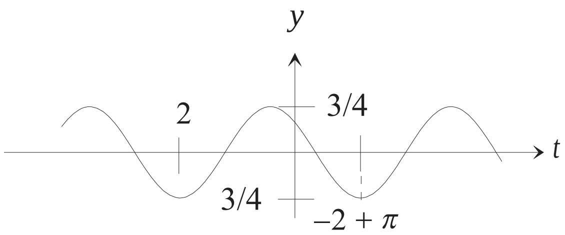
Solution: The graph of $y = 4\cos\left(-\frac{1}{2}t\right)$ has the same general appearance as the fundamental cosine function except that the period is $\dfrac{2\pi}{\left|-\frac{1}{2}\right|} = 4\pi$ units and the amplitude is 4.

HINT

To graph the function

$$y\left(t\right) = -\frac{3}{4}\cos\left(2t+4\right)$$

notice that it is the result of four changes in the fundamental cosine graph, and the function needs to be in the form $f\left(t\right) = A\cos\left[a\left(t+b\right)\right]+c$.



Solution: The function, $y(t) = -\frac{3}{4}\cos(2t+4)$ needs to be in the form $f(t) = A\sin[a(t+b)] + c$. So write,

$$y(t) = -\frac{3}{4}\cos(2t+4) = -\frac{3}{4}\cos[2(t+2)].$$

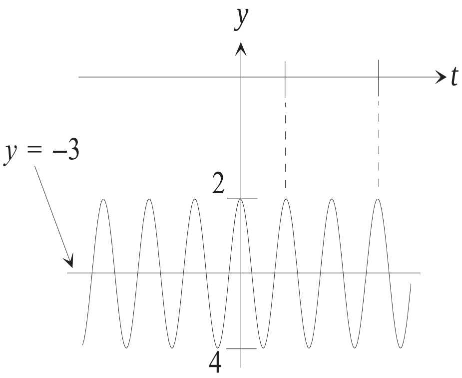
Then, y has a period of $\frac{2\pi}{|2|} = \pi$ and an amplitude of $\frac{3}{4}$. Also, y has a phase shift of 2 units to the left and should be reflected around the t-axis because of the negative sign preceding the amplitude. ◀

HINT

To graph the function

$$y(t) = \cos(6t) - 3$$

there is a change in the period and the vertical shift of the fundamental cosine graph.



Solution: The graph of $y = \cos(6t) - 3$ has the same general appearance as the fundamental cosine function except that the period is $\dfrac{2\pi}{|6|} = \dfrac{\pi}{3}$ units and there is a vertical shift down 3 units.

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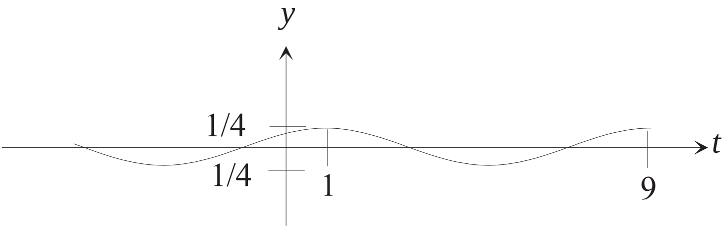
HINT

To graph the function

$$y\,(t) = \frac{1}{4}\cos\left[\frac{\pi}{4}\,(t-1)\right]$$

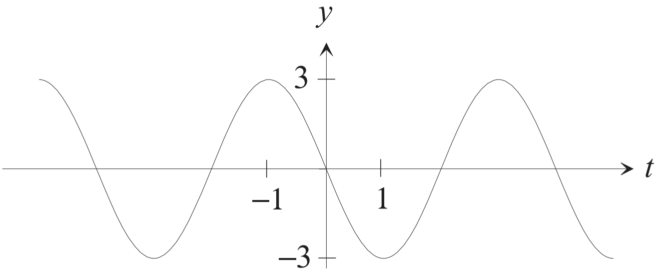
notice there is a change in the period, phase shift, and amplitude of the fundamental cosine graph.



Solution: The graph of $y = \frac{1}{4}\cos\left[\frac{\pi}{4}(t-1)\right]$ has the same general appearance as the fundamental cosine function except that the period is $\dfrac{2\pi}{\left|\frac{\pi}{4}\right|} = 8$ units, there is a phase shift of 1 unit to the right, and the amplitude is $\frac{1}{4}$ unit. ◀

HINT

To find a sine function that whose graph is given in the figure first determine the period.



Answer: $\boxed{y(t) = 2\sin\left(\frac{\pi}{2}t + \pi\right)}$

Solution: Since the graph of the function is symmetric with respect to the t-axis and deviates from this axis by 2 units, its amplitude is 2. Next, the distance measured along the t-axis from one peak to the next suggests that the period is 4 units. This means that $\frac{2\pi}{a} = 4$ so that $a = \frac{\pi}{2}$. Noticing differences between the given graph and the fundamental sine function, there is a phase shift of 2 units to the left. Using the form $f(t) = A\sin[a(t+b)] + c$ this indicates that

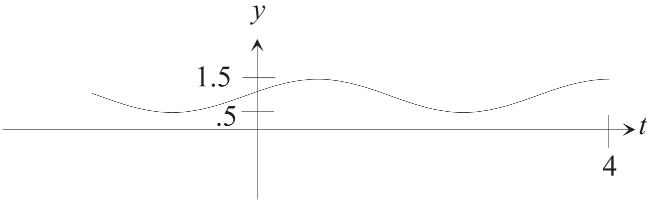
$$y(t) = 2\sin\left[\frac{\pi}{2}(t+2)\right] = 2\sin\left(\frac{\pi}{2}t + \pi\right).$$

There are other acceptable answers for this question including

$$y_a(t) = -2\sin\left[\frac{\pi}{2}(t+4)\right] \text{ and } y_b(t) = 2\sin\left[\frac{\pi}{2}(t-2)\right].$$

HINT

To find a sine function for the graph



note any changes from the fundamental sine graph.

Answer: $\boxed{y\left(t\right) = \frac{1}{2}\sin\left(\frac{1}{2}t\right) + 1}$

Solution: There is a vertical shift of 1 unit up since the graph is symmetric with respect to the line $y = 1$. The maximum deviation of $\frac{1}{2}$ from this line of symmmetry gives the amplitude of $\frac{1}{2}$ units. The distance measured along the t-axis from one peak to the next means the period is 4π units. Since $\frac{2\pi}{a} = 4\pi$ we have $a = \frac{2\pi}{4\pi} = \frac{1}{2}$. Hence,

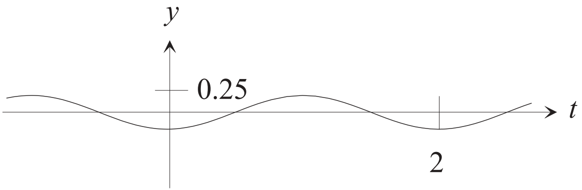
$$y\left(t\right) = \frac{1}{2}\sin\left(\frac{1}{2}t\right) + 1.$$

There are other acceptable answers for this question including

$$y_a\left(t\right) = -\frac{1}{2}\sin\left(\frac{t}{2} - \pi\right) + 1 \text{ and } y_b\left(t\right) = \frac{1}{2}\sin\left(\frac{t}{2} - 2\pi\right) + 1.$$

HINT

To find a cosine function for the graph



determine the length of one oscillation of the wave.

Answer: $\boxed{y(t) = \frac{1}{4}\cos(t+\pi)}$

Solution: Find a peak of the graph which is the maximum deviation from the t-axis, this gives the amplitude of $\frac{1}{4}$ units. Noticing differences between the given graph and the fundamental cosine graph, there is a phase shift of π units to the left. Using the form $f(t) = A\cos[a(t+b)] + c$ this indicates that

$$y(t) = \frac{1}{4}\cos(t+\pi).$$

There are other acceptable answers for this question including

$$y_a(t) = \frac{1}{4}\cos(t-\pi) \text{ and } y_b(t) = -\frac{1}{4}\cos t.$$

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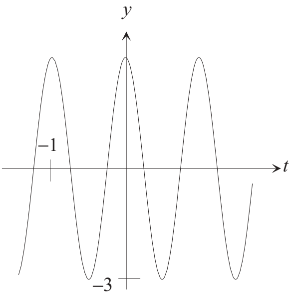
HINT



To find a cosine function for the given graph note any changes from the fundamental cosine graph in the period, phase shift, amplitude, or vertical shift.

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Answer: $\boxed{y(t) = 3\cos(2\pi t)}$

Solution: Find a peak of the graph which is the maximum deviation from the t-axis, this gives the amplitude of 3 units. Next, the distance measured along the t-axis from one peak to the next is the period of the function. In this case the period is 1 unit. This means that $\frac{2\pi}{a} = 1$ so that $a = 2\pi$.Using the form $f(t) = A\cos[a(t+b)] + c$ this indicates that

$$y(t) = 3\cos(2\pi t).$$

There are other acceptable answers for this question including

$$y_a(t) = -3\cos\left[2\pi\left(t + \frac{1}{2}\right)\right] \text{ and } y_b(t) = 3\cos[2\pi(t+1)].$$

◀

HINT

To find a cosine function whose graph is identical to the graph of the function $y(t) = 5\sin\left(\frac{1}{2}t + \frac{\pi}{2}\right)$ use an appropriate phase shift.

Answer: $\boxed{y(t) = 5\cos\left(\frac{1}{2}t\right)}$

Solution: The function, $y(t) = 5\sin\left(\frac{1}{2}t + \frac{\pi}{2}\right)$ needs to be in the form $f(t) = A\sin[a(t+b)] + c$. So write,

$$y(t) = 5\sin\left(\frac{1}{2}t + \frac{\pi}{2}\right) = 5\sin\left[\frac{1}{2}(t+\pi)\right].$$

The fundamental cosine function is the same basic shape as the fundamental sine function with a phase shift of $\frac{\pi}{2}$ units (one forth of the period) to the right. The period of the given function is $\dfrac{2\pi}{\left|\frac{1}{2}\right|} = 4\pi$ and one forth of the period is π. So, similarly shift the given function to get the given answer. There are other acceptable answers for this question including

$$y_a(t) = -5\cos\left[\frac{1}{2}(t+2\pi)\right] \text{ and } y_b(t) = 5\cos\left[\frac{1}{2}(t-4\pi)\right].$$

HINT

To find a sine function whose graph is identical to the graph of the function $y(t) = \frac{1}{2}\cos[\pi(t+1)] + 1$ use an appropriate phase shift.

Answer: $\boxed{y(t) = \frac{1}{2}\sin\left[\pi\left(t+\frac{3}{2}\right)\right]+1}$

Solution: The fundamental sine function is the same basic shape as the fundamental cosine function with a phase shift of $\frac{\pi}{2}$ units (one forth of the period) to the left. The period of the given function is $\dfrac{2\pi}{\pi} = 2$ and one forth of the period is $\frac{1}{2}$. So, similarly shift the given function to get the answer.

$$y(t) = \frac{1}{2}\sin\left[\pi\left(t+\frac{3}{2}\right)\right]+1$$

There are other acceptable answers for this question including

$$y_{a,b}(t) = \pm\frac{1}{2}\sin\left[\pi\left(t\mp\frac{1}{2}\right)\right]+1.$$

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