THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

*Flash Cards*

Right triangle trigonometry

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**Instructions:** Click on the Begin button to view the first randomly selected card. Click on FS to view the cards in full screen mode (works only outside a web browser). The Home button on the first page goes to the WebTrig home page; otherwise, the Home button returns to this page. The Close button closes the document (use outside a web browser).
A right triangle contains a 50° angle that has an adjacent side of length 6.3 units. Find the length of the opposite side and the hypotenuse.
A right triangle contains an 18° angle that has an opposite side of length 1.7 units. Find the length of the adjacent side and the hypotenuse.
A right triangle contains a 62° angle that has a hypotenuse of length 8.3 units. Find the length of the adjacent side and the opposite side.
Suppose \( \sin \alpha = \frac{6}{11} \). Without using a calculator find \( \cos \alpha \) and \( \tan \alpha \).
Suppose \( \cos \alpha = \frac{3}{5} \). Without using a calculator find \( \sin \alpha \) and \( \tan \alpha \).
Suppose \( \tan \alpha = \frac{5}{12} \). Without using a calculator find \( \sin \alpha \) and \( \cos \alpha \).
Let $\alpha$ denote a non-right angle of a right triangle. If the side adjacent $\alpha$ is 9 units long and the hypotenuse is 18 units, find $\sin \alpha$. 
Let $\alpha$ denote a non-right angle of a right triangle. If the side adjacent $\alpha$ is 12.2 units long and the hypotenuse is 30 units, find $\cot \alpha$. 
Let $\alpha$ denote a non-right angle of a right triangle. If the side opposite $\alpha$ is 12 units long and the hypotenuse is 24 units, find $\cos \alpha$. 
Let $\alpha$ denote a non-right angle of a right triangle. If the side adjacent $\alpha$ is 8 units long and the hypotenuse is 21 units, find $\csc \alpha$. 
Let $\alpha$ denote a non-right angle of a right triangle. If the side opposite $\alpha$ is 15 units long and the hypotenuse is 31 units, find sec $\alpha$. 
Let $\alpha$ denote a non-right angle of a right triangle. If the side adjacent $\alpha$ is 4.6 units long and the hypotenuse is 9.2 units, find $\tan \alpha$. 
Let $T$ be the triangle depicted in the figure. Find $\sin \alpha$. 

![Triangle Diagram]
Let $T$ be the triangle depicted in the figure. Find $\sin \gamma$. 

![Diagram of triangle with sides labeled $b$, $14$, $10$, and angles $72^\circ$, $\beta$, $\gamma$.]
Let $T$ be depicted in the figure below. Find $c$. 

![Diagram](image_url)
Let $T$ be the triangle in the figure. Find $a$. 

![Diagram of triangle with sides labeled 19.4, 27°, 112°, and unknown side labeled $a$.]
Let $T$ be the triangle in the figure below. Find $\cos \beta$. 

![Diagram of a triangle with sides labeled 18, 11, 9, and angles labeled $\gamma$, $\alpha$, and $\beta$.]
Let $T$ be the triangle depicted in the figure below. Find $\cos \gamma$. 

![Triangle Diagram]

$42 \quad \gamma \quad 45$

$\alpha \quad \beta \quad 70$
Let $T$ be the triangle given in the figure below. Find $a$. 

![Diagram of a triangle with sides 14, 24, and 47 and angles $\gamma$, $\alpha$, and $\beta$.]
Determine $b$ in the figure.
HINT

To find the length of the opposite side and the hypotenuse of a right triangle with a 50° angle that has an adjacent side of length 6.3 units, use the triangular definition of the cosine of an angle which is the length of the adjacent side divided by the hypotenuse.
Answer: \[ \text{hyp} = 9.8 \text{ and opp} = 7.51 \]

Solution: First, find the hypotenuse using the cosine ratio:

\[
\cos 50^\circ = \frac{6.3}{\text{hyp}}.
\]

So,

\[
\text{hyp} = \frac{6.3}{\cos 50^\circ} = 9.8011
\]

Next, the Pythagorean theorem suggests that

\[
(9.8011)^2 = (6.3)^2 + (\text{opp})^2.
\]

So,

\[
(\text{opp})^2 = 56.372
\]

Hence,

\[
\text{opp} = \sqrt{56.372} = 7.5081
\]

\[1(\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2\]
HINT

To find the length of the adjacent side and the hypotenuse of a right triangle with an $18^\circ$ angle that has an adjacent side of length 1.7 units, use the triangular definition of the sine of an angle which is the length of the opposite side divided by the hypotenuse.
Answer: \[ \text{hyp} = 5.5 \text{ and adj} = 5.23 \]

Solution: Find the hypotenuse using the sine ratio:

\[
\sin 18^\circ = \frac{1.7}{\text{hyp}}
\]

so,

\[
\text{hyp} = \frac{1.7}{\sin 18^\circ} = 5.5013
\]

Next, the Pythagorean theorem suggests that

\[
(5.5013)^2 = (\text{adj})^2 + (1.7)^2.
\]

So,

\[
(\text{adj})^2 = 27.374
\]

Hence,

\[
\text{adj} = \sqrt{27.374} = 5.232
\]
HINT

To find the length of the adjacent side and the opposite side of a right triangle with a $62^\circ$ angle that has an adjacent side of length 8.3 units, use the triangular definition of the sine of an angle which is the length of the opposite side divided by the hypotenuse.
Answer: \[ \text{opp} = 7.33 \text{ and } \text{adj} = 3.9 \]

Solution: Using the sine ratio we get

\[
\sin 62^\circ = \frac{\text{opp}}{8.3}.
\]

So,

\[
\text{opp} = 8.3 \cdot \sin 62^\circ \\
= 7.3285
\]

Next, the Pythagorean theorem suggests that

\[
(8.3)^2 = (\text{adj})^2 + (7.3285)^2
\]

so,

\[
(\text{adj})^2 = 15.183.
\]

Hence,

\[
\text{adj} = \sqrt{15.183} = 3.8965
\]

\[
(hyp)^2 = (\text{adj})^2 + (\text{opp})^2
\]
HINT

To find $\cos \alpha$ and $\tan \alpha$ when $\sin \alpha = \frac{6}{11}$

use the triangular definition of the sine of an angle which is $\sin \alpha = \frac{\text{opp}}{\text{hyp}}$. 
Solution: Given \( \sin \alpha = \frac{6}{11} \), use the Pythagorean theorem to get

\[(11)^2 = (\text{adj})^2 + (6)^2.\]

So, \((\text{adj})^2 = 85\), or

\(\text{adj} = \sqrt{85}.\)

Hence,

\[
\cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{85}}{11}
\]

and

\[
\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{6}{\sqrt{85}} = \frac{6\sqrt{85}}{85}.
\]
HINT

To find \( \sin \alpha \) and \( \tan \alpha \) when

\[
\cos \alpha = \frac{3}{5},
\]

use the triangular definition

\[
\cos \alpha = \frac{\text{adj}}{\text{hyp}}.
\]
Answer: $\sin \alpha = \frac{4}{5}$ and $\tan \alpha = \frac{4}{3}$

Solution: Given

$\cos \alpha = \frac{3}{5} = \frac{\text{adj}}{\text{hyp}}$,

and using the Pythagorean theorem we get

$(5)^2 = (3)^2 + (\text{opp})^2$.

Consequently,

$\text{opp} = \sqrt{16} = 4$

suggesting that

$\sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$

and

$\tan \alpha = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$. 

$5(\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2$.
HINT

To find $\sin \alpha$ and $\cos \alpha$ when

$$\tan \alpha = \frac{5}{12}$$

recall the triangular definitions of these functions.
Answer: \( \sin \alpha = \frac{5}{13} \) and \( \tan \alpha = \frac{12}{13} \)

Solution: Recall that

\[ \tan \alpha = \frac{5}{12} = \frac{\text{opp}}{\text{adj}}. \]

Next, find the hypotenuse using the Pythagorean theorem\(^6\)

\[ (\text{hyp})^2 = (12)^2 + (5)^2 = 169. \]

Consequently, \( \text{opp} = \sqrt{169} = 13 \)

so that

\[ \sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13} \]

and

\[ \cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13}. \]
HINT

To find $\sin \alpha$ if the side adjacent $\alpha$ is 9 units long and the hypotenuse is 18 units remember the triangular definition of the sine of an angle uses the length of the side opposite $\alpha$ and the length of the hypotenuse.
Answer: \( \sin \alpha = .87 \)

Solution: Find the opposite side using the pythagorean theorem \((\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2\)

\[
(18)^2 = (9)^2 + (\text{opp})^2
\]

\[\Rightarrow (\text{opp})^2 = 243\]

\[\Rightarrow \text{opp} = \sqrt{243} = 15.588\]

Then we know,

\[
\sin \alpha = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{243}}{18} = 0.86603.
\]
HINT

To find \( \cot \alpha \) if the side adjacent \( \alpha \) is 12.2 units long and the hypotenuse is 30 units recall that

\[
\cot \alpha = \frac{\text{adj}}{\text{opp}}
\]
Answer: \[ \cot \alpha = 0.45 \]

Solution: Find the opposite side using the pythagorean theorem \((\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2\)

\[
(30)^2 = (12.2)^2 + (\text{opp})^2
\]

\[\Rightarrow (\text{opp})^2 = 751.16\]

\[\Rightarrow \quad \text{opp} = \sqrt{751.16} = 27.407\]

We know

\[
\cot \alpha = \frac{\text{adj}}{\text{opp}} = \frac{12.2}{27.407} = 0.44514\]
HINT

To find $\cos \alpha$ if the side opposite $\alpha$ is 12 units long and the hypotenuse is 24 units use the triangular definition of the cosine of an angle which uses the length of the hypotenuse and side adjacent to $\alpha$. 
Answer: \[ \cos \alpha = .87 \]

Solution: Find the adjacent side using the pythagorean theorem \((\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2\)

\[(24)^2 = (\text{adj})^2 + (12)^2\]

\[\implies (\text{adj})^2 = 432\]

\[\implies \text{adj} = \sqrt{432} = 20.785\]

We know,

\[\cos \alpha = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{432}}{24} = \frac{\sqrt{3}}{2} = 0.86603\]
HINT

To find $\csc \alpha$ if the side adjacent $\alpha$ is 8 units long and the hypotenuse is 21 units use

$$\csc \alpha = \frac{\text{hyp}}{\text{opp}}$$
Answer: \( \csc \alpha = 1.08 \)

Solution: Find the opposite side using the pythagorean theorem \((\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2\)

\[
(21)^2 = (8)^2 + (\text{opp})^2
\]

\[
\implies (\text{opp})^2 = 377
\]

\[
\implies \text{opp} = \sqrt{377} = 19.416
\]

We know,

\[
\csc \alpha = \frac{\text{hyp}}{\text{opp}} = \frac{21}{\sqrt{377}} = 1.0816
\]
HINT

To find $\sec \alpha$ if the side opposite $\alpha$ is 15 units long and the hypotenuse is 31 units you will need the hypotenuse and the side adjacent to $\alpha$. 
Answer: \[\text{sec } \alpha = 1.14\]

Solution: Find the adjacent side using the pythagorean theorem \((\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2\)

\[
(31)^2 = (\text{adj})^2 + (15)^2
\]

\[\implies (\text{adj})^2 = 736
\]

\[\implies \text{adj} = \sqrt{736} = 27.129
\]

We know,

\[
\text{sec } \alpha = \frac{\text{hyp}}{\text{adj}} = \frac{31}{\sqrt{736}} = 1.1427
\]
HINT

To find $\tan \alpha$ if the side adjacent $\alpha$ is 4.6 units long and the hypotenuse is 9.2 units you will need the length of the side opposite $\alpha$ and the hypotenuse.
Answer: \[ \tan \alpha = 1.73 \]

Solution: Find the opposite side using the pythagorean theorem.

\[
(hyp)^2 = (adj)^2 + (opp)^2
\]
\[
\Rightarrow (9.2)^2 = (4.6)^2 + (opp)^2
\]
\[
\Rightarrow (opp)^2 = 63.48
\]
\[
\Rightarrow opp = \sqrt{63.48} = 7.9674
\]

We know,

\[
\tan \alpha = \frac{opp}{adj} = \frac{7.9674}{4.6} = 1.732
\]
HINT

To find \( \sin \alpha \) if \( a = 9 \), \( b = 10 \), and \( \beta = 67^\circ \) as depicted in the figure, use the law of sines.
Answer: $\sin \alpha = 0.83$

Solution: By the law of sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b}$$

$$\frac{\sin \alpha}{9} = \frac{\sin 67^\circ}{10}$$

so

$$\sin \alpha = 9 \times 0.09205 = 0.82845$$
HINT

To find $\sin \gamma$ in the given triangle recall that the law of sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}.$$
Answer: \[ \sin \gamma = 0.07 \]

Solution: Use the law of sines
\[
\frac{\sin \alpha}{a} = \frac{\sin \gamma}{c}
\]
to obtain
\[
\frac{\sin 72^\circ}{14} = \frac{\sin \gamma}{10}
\]
\[
\sin \gamma = 10 \times 0.00679
\]
\[
= 0.0679
\]
HINT

To find $c$ if $\beta = 26^\circ$, $b = 7$, and $\gamma = 75^\circ$ as depicted in the figure below use the law of sines.
Answer: \( c = 15.42 \)

Solution: By the law of sines

\[
\frac{\sin \gamma}{c} = \frac{\sin \beta}{b}.
\]

So

\[
\frac{\sin 75^\circ}{c} = \frac{\sin 26^\circ}{7},
\]

\[
7 \sin 75^\circ = c \sin 26^\circ
\]

\[
c = \frac{7 \sin 75^\circ}{\sin 26^\circ} = 15.424
\]
To find $a$ in the figure use the law of sines which equates $\frac{\sin \alpha}{a}$ to similar ratio.
Answer: \[ \alpha = 9.5 \]

Solution: By the law of sines

\[ \frac{\sin \alpha}{a} = \frac{\sin \beta}{b}. \]

So

\[ \frac{\sin 27^\circ}{a} = \frac{\sin 112^\circ}{19.4} \]

\[ 19.4 \sin 27^\circ = a \sin 112^\circ \]

\[ \alpha = \frac{19.4 \sin 27^\circ}{\sin 112^\circ} = 9.499 \]
HINT

To find $\cos \beta$ for the angle $\beta$ in the triangle, use the law of cosines. Remember this law can be written in three different forms.
Answer: \( \cos \beta = .88 \)

Solution: By the law of cosines \( (b^2 = a^2 + c^2 - 2ac \cos \beta) \)

\[
11^2 = 18^2 + 9^2 - 2 \times 18 \times 9 \cos \beta
\]

\[
11^2 = 405 - 324 \cos \beta
\]

\[
\Rightarrow \cos \beta = \frac{121 - 405}{-324} = 0.87654
\]
HINT

To find $\cos \gamma$ if $a = 45$, $b = 42$, and $c = 70$ recall the law of cosines:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma.$$
Answer: $\cos \gamma = -0.29$

Solution: Using the law of cosines $c^2 = a^2 + b^2 - 2ab \cos \gamma$ we have

$$70^2 = 45^2 + 42^2 - 2(45)(42)\cos \gamma$$

$$= 3789 - 3780 \cos \gamma$$

$$\Rightarrow \cos \gamma = \frac{4900 - 3789}{-3780} = -0.29392$$
HINT

To find $a$ if $\alpha = 47^\circ$, $b = 14$, and $c = 24$ use the law of cosines which equates

$$b^2 + c^2 - 2bc \cos \alpha$$

to another value.

![Diagram of a triangle with sides 14, 24, and 47 degrees]
Answer: $a = 17.71$

Solution: By the law of cosines $a^2 = b^2 + c^2 - 2bc \cos \alpha$ we have

$$a^2 = 14^2 + 24^2 - 2(14)(24) \cos 47^\circ$$

$$a^2 = 772 - 672 \cos 47^\circ$$

$$a^2 = 313.7$$

$$\implies a = \sqrt{313.7} = 17.712$$
To find $b$ in the triangle use the form of the law of cosines that equates $b^2$ to another value.
Answer: \( b = 2.67 \)

Solution: By the law of cosines

\[
b^2 = a^2 + c^2 - 2ac \cos \beta
\]

we have

\[
b^2 = 2^2 + (1.7)^2 - 2(2)(1.7) \cos 92^\circ
\]

\[
b^2 = 6.89 - 6.8 \cos 92^\circ
\]

\[
b^2 = 7.1273
\]

\[\implies b = \sqrt{7.1273} = 2.6697\]