THE UNIVERSITY OF AKRON
Theoretical and Applied Mathematics

Flash Cards

The Trigonometric Functions

by
Katie Jones
and
Tom Price

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WebTrig Flash Cards
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Find the $\sin t$ if $\cos t = \frac{6}{7}$ and

$$0 \leq t \leq \pi.$$

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Find the $\sin t$ if $0 \le t \le \pi$ and $\left(\frac{1}{3},\ y\right)$ is the point on the unit circle determined by the standard position angle of measure t rad.

Find $\cos t$ if $\left(x, \frac{1}{4}\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for

$$0 \leq t \leq \pi.$$

Find the $\cot t$ if $\left(x,\ \frac{3}{4}\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for

$$\frac{\pi}{2} \leq t \leq \pi.$$

Hint Soln
Next Home

Find the $\sec t$ if $\left(x, -\frac{2}{3}\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for

$$-\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

Hint Soln
Next Home

Find the $\csc t$ if $(-\frac{4}{5}, y)$ is the point on the unit circle determined by the standard position angle measuring t rad for

$$\pi \leq t \leq \frac{3\pi}{2}.$$

Find the $\tan t$ if $\left(-\frac{1}{4},\ y\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for

$$-\pi/2 < t \leq \pi.$$

Find the $\sin t$ at the radian measure

$$t = 27\pi.$$

Hint Soln
Next Home

Find the $\cot t$ at the radian measure

$$t = \frac{21\pi}{6}.$$

Find the point on the unit circle determined by the standard position angle measuring

$$t = \frac{11\pi}{2}\,\text{rad}\,.$$

Find the point on the unit circle determined by the standard position angle measuring

$$t = \frac{\pi}{3}\,\text{rad}\,.$$

Prove that

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

Hint Soln
Next Home

Find the $\csc \frac{4\pi}{3}$ without using a calculator.

Give all the radian measures

$$0 \leq t \leq 2\pi$$

where $\tan t$ is not defined. Why?

Give all the radian measures

$$0 \le t < 2\pi$$

where $\csc t$ is not defined. Why?

If $\sin t = b$ determine the value of $\csc(-t)$.

Hint Soln
Next Home

If the point on the unit circle determined by the standard position angle measuring t rad is (a, b), find $\cos(-t)$.

If the point on the unit circle determined by the standard position angle measuring t rad is (a, b), find $\tan(-t)$.

What is the period of $\sin t$.

What is the period of a $\tan t$.

HINT

To find the $\sin t$ if $\cos t = \frac{6}{7}$ and $0 \le t \le \pi$ recall that the $\sin t$ is defined to be equal to the value of the y-coordinate of the point on the unit circle determined by the standard position angle of measure t rad.

$$\text{Answer: } \boxed{\sin t = \frac{\sqrt{13}}{7}}$$

Solution: First we will determine the value of y using the equation of the unit circle $x^2 + y^2 = 1$:

$$\begin{aligned} \left(\frac{6}{7}\right)^2 + y^2 &= 1 \\ \Longrightarrow \qquad y^2 &= \frac{13}{49} \\ \Longrightarrow \qquad y &= \pm\sqrt{\frac{13}{49}} = \pm\frac{\sqrt{13}}{7}. \end{aligned}$$

Choose the positive answer for y since the sine function is positive for $0 \le t \le \pi$. Then, $\sin t = y = \frac{1}{7}\sqrt{13}$. ◀

HINT

To find the $\sin t$ if $0 \leq t \leq \pi$ and $\left(\frac{1}{3},\ y\right)$ recall that the $\sin t$ is defined to be equal to the value of the y-coordinate of the point on the unit circle determined by the standard position angle of measure t rad.

Answer: $\boxed{\sin t = \frac{2\sqrt{2}}{3}}$

Solution: First we determine the value of y using the equation of the unit circle $x^2 + y^2 = 1$:

$$\left(\frac{1}{3}\right)^2 + y^2 = 1$$

$$\Longrightarrow \qquad y^2 = \frac{8}{9}$$

$$\Longrightarrow \qquad y = \pm\frac{2\sqrt{2}}{3}.$$

Choose the positive answer for y since the sine function is positive for $0 \le t \le \pi$. Then, $\sin t = y = \frac{2}{3}\sqrt{2}$. ◀

HINT

To find the $\cos t$ if $\left(x, \frac{1}{4}\right)$ is the point on the unit circle determined by the angle measuring t rad for $0 \leq t \leq \pi$ recall that the $\cos t$ is defined to be equal to the value of the x-coordinate of the point on the unit circle determined by the standard position angle of measure t rad.

Answer: $\boxed{\cos t = \dfrac{\sqrt{15}}{4}}$

Solution: First we determine the value of x using the equation of the unit circle is $x^2 + y^2 = 1$:

$$
\begin{aligned}
x^2 + \left(\frac{1}{4}\right)^2 &= 1 \\
\Longrightarrow \qquad x^2 &= \frac{15}{16} \\
\Longrightarrow \qquad x &= \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}.
\end{aligned}
$$

Choose the positive answer for y since the cosine function is positive for $0 \le t \le \pi$. Then, $\cos t = x = \frac{1}{4}\sqrt{15}$. ◀

HINT

To find the $\cot t$ if $\left(x,\ \frac{3}{4}\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for $\frac{\pi}{2} \leq t \leq \pi$ recall that

$$\cot t = \frac{x}{3/4}.$$

Answer: $\boxed{\cot t = -\dfrac{\sqrt{7}}{3}}$

Solution: First we determine the value of x. The equation of the unit circle is $x^2 + y^2 = 1$. So,

$$x^2 + \left(\frac{3}{4}\right)^2 = 1$$

$$\Longrightarrow \qquad x^2 = \frac{7}{16}$$

$$\Longrightarrow \qquad x = \pm\frac{\sqrt{7}}{4}.$$

Choose the negative answer for x since the cotangent function is negative for $\frac{\pi}{2} \leq t \leq \pi$. Then, $\cot t = \dfrac{x}{y} = -\frac{1}{4}\sqrt{7} \cdot \frac{4}{3} = -\frac{1}{3}\sqrt{7}$.

◀

HINT

To find the $\sec t$ if $\left(x,\ -\frac{2}{3}\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ recall that

$$\sec t = \frac{1}{\cos t}.$$

Answer: $\boxed{\sec t = \dfrac{3\sqrt{5}}{5}}$

Solution: First we determine the value of x. The equation of the unit circle is $x^2 + y^2 = 1$. So,

$$x^2 + \left(-\frac{2}{3}\right)^2 = 1$$

$$\Longrightarrow \quad x^2 = \frac{5}{9}$$

$$\Longrightarrow \quad x = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}.$$

Choose the positive answer for x since the secant function is positive whenever $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$. Then, $\sec t = \dfrac{1}{x} = \dfrac{1}{\frac{1}{3}\sqrt{5}} = \frac{3}{5}\sqrt{5}$.

◀

HINT

To find the $\csc t$ if $\left(-\frac{4}{5}, y\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for $\pi \leq t \leq \frac{3\pi}{2}$ recall that the $\csc t$ is defined to be equal to $\dfrac{1}{\sin t}$.

Answer: $\boxed{\csc t = -\frac{5}{3}}$

Solution: First we determine the value of y. The equation of the unit circle is $x^2 + y^2 = 1$. So,

$$\left(-\frac{4}{5}\right)^2 + y^2 = 1$$

$$\Longrightarrow \qquad y^2 = \frac{9}{25}$$

$$\Longrightarrow \qquad y = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}.$$

Choose the negative answer for y since t lies in the third quadrant and the cosecant function is negative there. Then, $\csc t = \dfrac{1}{y} = \dfrac{1}{-\frac{3}{5}} = -\frac{5}{3}$.

HINT

To find the $\tan t$ if $\left(-\frac{1}{4},\ y\right)$ is the point on the unit circle determined by the standard position angle measuring t rad for $-\pi/2 < t \leq \pi$ recall that the $\tan t$ is defined to be equal to $\dfrac{\sin t}{\cos t}$.

Answer: $\boxed{\tan t = -\sqrt{15}}$

Solution: First we determine the value of y. The equation of the unit circle is $x^2 + y^2 = 1$. So,

$$\left(-\frac{1}{4}\right)^2 + y^2 = 1$$

$$\Longrightarrow \qquad y^2 = \frac{15}{16}$$

$$\Longrightarrow \qquad y = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}.$$

Choose the positive answer for y since the tangent function is negative whenever $-\pi/2 < t \le \pi$. Then, $\tan t = \dfrac{\sin t}{\cos t} = \dfrac{y}{x} = \dfrac{\frac{1}{4}\sqrt{15}}{-\frac{1}{4}} = -\sqrt{15}$.

◀

HINT

To find the $\sin t$ at the radian measure $t = 27\pi$ recall that the $\sin t$ is defined to be equal to the y-coordinate at the point on the unit circle determined by the standard position angle of radian measure t.

Answer: $\boxed{\sin 27\pi = 0}$

Solution: 27π rad is coterminal with an angle measuring π rad since $27\pi = 13\,(2\pi) + \pi$. The point on the unit circle determined by a standard position angle π rad wide is $(-1, 0)$, so $x = -1$ and $y = 0$. Then, $\sin t = y = 0$. ◀

HINT

To find the $\cot t$ at the radian measure $\frac{21\pi}{6}$ recall that the $\cot t$ is defined to be equal to $\dfrac{\cos t}{\sin t}$.

Answer: $\boxed{\cot \frac{21\pi}{6} = 0}$

Solution: An angle $\frac{21\pi}{6}$ rad wide is coterminal with an angle measuring $\frac{3\pi}{2}$ rad since $\frac{21\pi}{6} = 2\pi + \frac{3\pi}{2}$. TThe point on the unit circle determined by a standard position angle $\frac{3\pi}{2}$ rad wide is $(0, -1)$, so $x = 0$ and $y = -1$. Then, $\cot t = \dfrac{\cos t}{\sin t} = \dfrac{x}{y} = \dfrac{0}{-1} = 0$.

◀

HINT

To find the point on the unit circle determined by the standard position angle measuring $t = \dfrac{11\pi}{2}$ rad first find an angle coterminal to t which is between 0 rad and 2π rad.

Answer: $\boxed{(0, -1)}$

Solution: An angle $\frac{11\pi}{2}$ rad wide is coterminal with an angle measuring $\frac{3\pi}{2}$ rad since $\frac{11\pi}{2} = 2 \cdot 2\pi + \frac{3\pi}{2}$. The point on the unit circle determined by the standard position ange $\frac{3\pi}{2}$ rad wide is $(0, -1)$.

HINT

To find the point on the unit circle determined by the standard position angle measuring $t = \dfrac{\pi}{3}$ rad draw an equilateral triangle and use the fact that the radius of the unit circle is 1.

Answer: $\boxed{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$

Solution: Let (x, y) denote the point on the unit circle determined by the standard position angle $\frac{\pi}{3}$ rad . The points (x, y), $(1, 0)$, and $(0, 0)$ determine an equilateral triangle with side length 1 unit. This is verified since there are $180°$ in a triangle and after converting $\frac{\pi}{3}$ rad $= 60°$. Dropping an altitude from the point (x, y) to the x-axis, we get that the x-coordianate at the point is $\frac{1}{2}$. Then using the equation of the unit circle $x^2 + y^2 = 1$;

$$\left(\tfrac{1}{2}\right)^2 + y^2 = 1 \Longrightarrow y^2 = 1 - \frac{1}{4} \Longrightarrow y = \pm\tfrac{\sqrt{3}}{2}$$

Choose the positive value for y since $t = \dfrac{\pi}{3}$ rad is in the first quadrant. Then the point is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

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HINT

To prove that $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ without a calculator notice that the angle of measure $\pi/4$ rad can be used to form a right triangle in the first quadrant of the coordinate system.

Proof: Let (x, y) be the point on the unit circle determined by the standard position angle of measure $\frac{\pi}{4}$ rad $= 45\,°$. The triangle formed by the points (x, y), $(0, 0)$, and $(1, 0)$ has angles $45\,°, 45\,°$, and $90\,°$, which means the two legs formed by radii of the circle will have equal length. The equation of the unit circle is $x^2 + y^2 = 1$. So setting $x = y$, we get

$$
\begin{aligned}
& 2x^2 = 1 \\
\Longrightarrow \quad & x^2 = \tfrac{1}{2} \\
\Longrightarrow \quad & x = \frac{1}{\sqrt{2}}
\end{aligned}
$$

Then, $\cos t = x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. ◀

HINT

To find $\csc\frac{4\pi}{3}$ without using a calculator recall that

$$\csc(t+\pi) = -\csc t.$$

Answer: $\boxed{\csc t = -\frac{2}{\sqrt{3}}}$

Solution: Since $\csc(t+\pi) = -\csc t$ we have

$$\csc\frac{4\pi}{3} = \csc\left(\frac{\pi}{3}+\pi\right) = -\csc\frac{\pi}{3} = -\frac{2}{\sqrt{3}}.$$

HINT

To find all the radian measures

$$0 \leq t \leq 2\pi$$

where $\tan t$ is not defined recall that

$$\tan t = \frac{\sin t}{\cos t}.$$

Answer: $\boxed{\dfrac{\pi}{2}\text{ rad and }\dfrac{3\pi}{2}\text{ rad}}$

Solution: $\tan t = \dfrac{\sin t}{\cos t}$. Since you cannot divide by 0, the tangent function is undefined whenever $\cos t = 0$. This occurs when the x-coordinate at the point determined by t rad is 0, since $\cos t = x$. It follows that this happens whenever the point determined by the standard position angle t rad lies on the y-axis, so the radian measures are $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$. ◀

HINT

To find all the radian measures

$$0 \leq t < 2\pi$$

where $\csc t$ is not defined recall that the $\csc t$ is defined to be equal to $\dfrac{1}{\sin t}$.

Answer: $\boxed{0\,\text{rad and } \pi\,\text{rad}}$

Solution: $\csc t = \dfrac{1}{\sin t}$. Since you cannot divide by 0, the cosecant function is undefined whenever $\sin t = 0$. This occurs when the y-coordinate at the point determined by $t\,$rad is 0, since $\sin t = y$. It follows that this happens whenever this point lies on the x-axis, so the radian measures are 0 and π.

◀

HINT

To determine the value of $\csc(-t)$ given that $\sin t = b$ recall that

$$\sin(-t) = -\sin t$$

Answer: $\boxed{\csc(-t) = -\frac{1}{b}}$

Solution:

$$\csc(-t) = \frac{1}{\sin(-t)} = \frac{1}{-\sin t} = -\frac{1}{b}.$$

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Hint Soln
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HINT

To find $\cos(-t)$ given that the point on the unit circle determined by the standard position angle measuring t rad is (a, b), recall that

$$\cos(-t) = \cos t$$

Answer: $\boxed{\cos(-t) = a}$

Solution:

$$\cos(-t) = \cos t = b$$

HINT

To find $\tan(-t)$ given that the point on the unit circle determined by the standard position angle measuring t rad is (a, b), recall that

$$\tan(-t) = -\tan t.$$

Answer: $\boxed{\tan(-t) = \dfrac{-b}{a}}$

Solution:

$$\tan(-t) = -\tan t = -\frac{b}{a}.$$

HINT

To determine the period of $\sin t$ think about when the function has the same values or repeats itself.

Answer: $\boxed{2\pi}$

HINT

To determine the period of a $\tan t$ think about when the function has the same values or repeats itself.

Answer: $\boxed{\pi}$