1 Lesson 16: Identities and determinants

1.1 Identity Matrix

Definition 1 *The $n \times n$ identity matrix I_n is the matrix whose diagonal entries are all 1 and the remaining entries are 0.*

Example 2 *The 6×6 identity matrix is given below*

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 3 *Let A be an $n \times n$ matrix. Define $A^2 = AA$. More generally, set $A^k = \overbrace{A \cdot A \cdot \cdots \cdot A}^{k \text{ copies of } A}$ for any positive integer k. Prove that for any positive integer n and $k = 2, 3, 4$*

$$I_n^k = I_n$$

Does this suggest a proof for any positive integer k?

Exercise 4 *Let A be an 2×2 matrix. Prove prove that*

$$I_2 A = A = A I_2$$

Let A be any $m \times n$ matrix. Can you prove the more general result that

$$I_m A = A I_n = A$$

1.2 Determinants

Definition 5 *Let $A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix}$ where $n > 2$ and recall that $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$. Define*

$$\det A = a_{11} \begin{vmatrix} a_{2,2} & \cdots & a_{2,n} \\ \vdots & \ddots & \vdots \\ a_{n,2} & \cdots & a_{n,n} \end{vmatrix} - a_{1,2} \begin{vmatrix} a_{2,1} & a_{2,3} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,3} & \cdots & a_{n,n} \end{vmatrix} + \cdots + (-1)^{1+n} a_{1,n} \begin{vmatrix} a_{2,1} & a_{2,2} & \cdots & a_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n-1} \end{vmatrix}$$

Example 6 *Let* $A = \begin{bmatrix} 8 & -6 & -8 & -7 \\ -9 & -7 & 0 & 0 \\ 8 & 9 & 6 & -5 \\ 5 & 0 & 7 & 7 \end{bmatrix}$. *Then*

$$\begin{aligned} \det A &= 8\begin{vmatrix} -7 & 0 & 0 \\ 9 & 6 & -5 \\ 0 & 7 & 7 \end{vmatrix} - (-6)\begin{vmatrix} -9 & 0 & 0 \\ 8 & 6 & -5 \\ 5 & 7 & 7 \end{vmatrix} + (-8)\begin{vmatrix} -9 & -7 & 0 \\ 8 & 9 & -5 \\ 5 & 0 & 7 \end{vmatrix} - (-7)\begin{vmatrix} -9 & -7 & 0 \\ 8 & 9 & 6 \\ 5 & 0 & 7 \end{vmatrix} \\ &= 8\left(-7\begin{vmatrix} 6 & -5 \\ 7 & 7 \end{vmatrix} - 9\begin{vmatrix} 0 & 0 \\ 7 & 7 \end{vmatrix} + 0\begin{vmatrix} 0 & 0 \\ 6 & -5 \end{vmatrix}\right) \\ &\quad +6\left((-9)\begin{vmatrix} 6 & -5 \\ 7 & 7 \end{vmatrix}\right) \\ &\quad -8 \ \cdots \end{aligned}$$

Exercise 7 *Complete the calculations for the above example.*

Example 8 *Now we will jump to my target 1.1*

2 Project 16

Instructions: Create a file containing the answers to the exercises in this lesson. You do not need to include the definitions and examples. Submit a .tex version of your file to teprice@uakron.edu. The name of your files should be of the form **yourlastname16.tex**. All calculations should be done using the CAS in SWP.