# 1 Lesson 14: More on matrices III

## 1.1 The reshape command

Consider the sequence:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0$$

$$, \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 8 & 7 & 6 \\ 5 & 4 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$ ,We will use the reshape command to obtain the matrix

**Exercise 1** *Reshape the above sequence to* $n$ *columns where* $n = 1, 2, 5, 10, 17, 18$

**Exercise 2** *Reshape the above sequence to* 2 *columns to obtain*

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \\ 9 & 8 \\ 7 & 6 \\ 5 & 4 \\ 3 & 2 \\ 1 & 0 \end{bmatrix}$$

*Next, reshape the above* $9 \times 2$ *matrix to*

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{bmatrix}$$

## 1.2 Matrix addition

Two matrices of the same dimension may be summed by adding their corresponding entries. This is called matrix addition.

**Example 3**

$$\begin{bmatrix} -348 & 470 & -608 \\ -686 & 702 & -61 \\ -49 & -433 & 966 \\ 902 & -942 & 712 \end{bmatrix} + \begin{bmatrix} 597 & -245 & 79 \\ 747 & 477 & -535 \\ -906 & -905 & -266 \\ -8 & 765 & 448 \end{bmatrix}$$

$$: \begin{bmatrix} 249 & 225 & -529 \\ 61 & 1179 & -596 \\ -955 & -1338 & 700 \\ 894 & -177 & 1160 \end{bmatrix}$$

**Exercise 4** *Generate two $2 \times 2$ matrices of your choice and add them.*

**Exercise 5** *Let* $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ *and* $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$. *Prove that* $A + B = B + A$. *That is, matrix addition is communicative. This result holds for two* $n \times n$ *matrices.*

**Example 6** *This example illustrates that matrix addition is associative (i.e.,* $(A + B) + C = A + (B + C)$*). Consider*

$$\begin{aligned}
&\left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right) + \begin{bmatrix} i & j \\ k & l \end{bmatrix} \\
= &\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix} \\
= &\begin{bmatrix} (a+e)+i & (b+f)+j \\ (c+g)+k & (d+h)+l \end{bmatrix} \\
= &\begin{bmatrix} a+(e+i) & b+(f+j) \\ c+(g+k) & d+(h+l) \end{bmatrix} \\
= &\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e+i & f+j \\ g+k & h+l \end{bmatrix} \\
= &\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \left( \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix} \right)
\end{aligned}$$

## 1.3 Scalar multiplication

A scalar for a matrix with real entries is a real number. If the matrix allows complex entries, the scalar may be a complex number. A scalar times a matrix $A = (a_{i,j})$ results in multiplying each entry of the matrix by the scalar so that

$$c\,(a_{i,j}) = (ca_{i,j})$$

Hence,

$$c \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix} = \begin{bmatrix} ca_{1,1} & ca_{1,2} & c\cdots & ca_{1,n} \\ ca_{2,1} & ca_{2,2} & c\cdots & ca_{2,n} \\ c\vdots & c\vdots & c\ddots & c\vdots \\ ca_{m,1} & ca_{m,2} & c\cdots & ca_{m,n} \end{bmatrix}$$

$$(a_{i,j}) + (b_{i,j}) = (a_{i,j} + b_{i,j}) = (b_{i,j} + a_{i,j}) = (b_{i,j}) + (a_{i,j})$$

**Example 7** *Scalar multiplication using complex entries:*

$$i \begin{bmatrix} 2 & 1-i \\ i & 2+1 \end{bmatrix} = \begin{bmatrix} 2i & 1+i \\ -1 & 3i \end{bmatrix}$$

Fill in the missing entries below

$$-7 \begin{array}{cc} 761 & -892 \\ -564 & -826 \\ 251 & -414 \end{array} = \begin{array}{cc} -5327 & 6244 \\ 3948 & 5782 \\ & 2898 \end{array}$$

**Exercise 8** *Let $A$ be a $2 \times 2$ real matrix. Show that for any number (scalar) $c$*

$$\det(cA) = c^2 \det A$$

*What result would hold if $A$ is an $n \times n$ real matrix?*

**Exercise 9** *Use the definitions of matrix addition and scalar multiplication to define matrix difference. (You may recall that the difference of two real numbers $a$ and $b$ is defined by $a + (-1)\, b$). Specifically, if $A$ and $B$ are two matrices of the same dimension, define $A - B$.*

## 1.4 The inner product

The inner product (or dot product) of a $1 \times n$ vector with an $n \times 1$ vector produces a scalar by multiplying corresponding entries and summing. That is,

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n = \sum_{j=1}^{n} a_j b_j$$

**Example 10**

$$\begin{bmatrix} 2 & 4 & -1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ 3 \\ 5 \end{bmatrix} = -4 + 16 - 3 + 0 = 9$$

**Exercise 11** *Compute the innerproduct*

$$\begin{bmatrix} 1 & 3 & -2 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 4 \\ 5 \end{bmatrix}$$

**Exercise 12** *Fill in the blank.*

$$\begin{bmatrix} 1 & 0 & -2 & \quad & 2 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 0 \\ 5 \\ -2 \\ -3 \end{bmatrix} = -33$$

**Exercise 13** *Prove that the inner product of any real $1 \times n$ vector with the $n \times 1$ zero vector is the scalar 0.*

## 2 Project 14

Instructions: Create a file containing the answers to the exercises in this lesson. You do not need to include the definitions and examples. Submit a .tex version of your file to teprice@uakron.edu. The name of your files should be of the form **yourlastname13.tex**. All calculations should be done using the CAS in SWP.