Lesson 6: More calculations

In this lesson we will use the project exercises to present additional strategies for using the CAS in SWP.

1 Project 6:

Instructions: Create a file containing the items below and their solutions. Submit .tex a version of your file to teprice@uakron.edu. The name of your files should be of the form **yourlastname06.tex** All calculations should be done using the CAS in SWP.

Exercise 1 *Compute:* $\lim_{h\to 0}\frac{h}{1-\sqrt{1+h}}$ *(On this exercise you should include all the details below.)*

Solution 2 *Consider*

$$\begin{aligned}\lim_{h\to 0}\frac{h}{1-\sqrt{1+h}} &= \lim_{h\to 0}\left[\frac{h}{1-\sqrt{1+h}}\left(\frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}\right)\right] \\ &= \lim_{h\to 0}\frac{h\left(1+\sqrt{1+h}\right)}{\left(1-\sqrt{1+h}\right)\left(1+\sqrt{1+h}\right)} \\ &= \lim_{h\to 0}\frac{h\left(1+\sqrt{1+h}\right)}{1-\left(\sqrt{1+h}\right)^2} \\ &= \lim_{h\to 0}\frac{1+\sqrt{1+h}}{-1} \\ &= \frac{1+\sqrt{1+0}}{-1} = -2\end{aligned}$$

Exercise 3 *Compute:* $\lim_{x\to 1}\left(\frac{x+2}{x-1}-\frac{x^2+5}{x^2-1}\right)$

Solution 4 *Solution:*

$$\begin{aligned}
\lim_{x\to 1}\left(\frac{x+2}{x-1}-\frac{x^2+5}{x^2-1}\right) &= \lim_{x\to 1}\left(\frac{x+2}{x-1}-\frac{x^2+5}{(x-1)(x+1)}\right)\\
&= \lim_{x\to 1}\frac{(x+2)(x+1)-x^2-5}{(x-1)(x+1)}\\
&= \lim_{x\to 1}\frac{x^2+3x+2-x^2-5}{(x-1)(x+1)}\\
&= \lim_{x\to 1}\frac{3x-3}{(x-1)(x+1)}\\
&= \lim_{x\to 1}\frac{3(x-1)}{(x-1)(x+1)}\\
&= 3\lim_{x\to 1}\frac{1}{(x+1)}\\
&= 3\lim_{x\to 1}\frac{1}{(x+1)} = 3\left(\frac{1}{2}\right) = \frac{3}{2}
\end{aligned}$$

Exercise 5 *Compute:* $\lim_{x\to 2^+}\frac{x+2}{x-2}$

Solution 6 *Let* $t = x-2$ *and consider*

$$\begin{aligned}
\lim_{x\to 2^+}\frac{x+2}{x-2} &= 4\lim_{x\to 2^+}\frac{1}{x-2}\\
&= 4\lim_{t\to 0^+}\frac{1}{t} \quad t = x-2\\
&= \infty
\end{aligned}$$

Exercise 7 *Solve* $3x^3+2x+1=0$

Exercise 8 *Solve exact*

$$\begin{aligned}
a^2-b^2 &= 5\\
a+b &= 1
\end{aligned}$$

Exercise 9 *Solve*

$$\begin{aligned}
a^2-b^2 &= 5\\
a+c &= 1
\end{aligned}$$

Exercise 10 *Combine*

$$2^a 2^b =$$

Exercise 11 *Expand*

$$\sin(a+b)$$

and

$$\sin(12\theta) =$$

Exercise 12 *Simplify and Factor*

$$\frac{a^4 - b^4}{a - b}$$

Exercise 13 π

Exercise 14 *Solve the following differential equation*

$$\frac{dy}{dx} = xy$$

Exercise 15 *Solve the following differential equation*

$$y'' + y = x^2$$

Example 16 *(Graduate - and other interested - Students) The wave equation*

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

has exact solution $y\left(t, x\right) = F_1\left(-x - at\right) + F_2\left(x - at\right)$ *where* F_1 *and* F_2 *are functions of a real variable. To see this note that*

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2} &= \frac{\partial}{\partial t}\frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t}\left(-aF_1'\left(-x - at\right) - aF_2'\left(x - at\right)\right) \\ &= a^2\left(F_1''\left(-x - at\right) + F_2''\left(x - at\right)\right) \end{aligned}$$

Likewise,

$$a^2 \frac{\partial^2 y}{\partial x^2} = a^2\left(F_1''\left(-x - at\right) + F_2''\left(x - at\right)\right)$$

and it follows that

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

As a specific example set

$$F_1(x) = \sin\left(\pi x\right) + \sin\left(3\pi x\right)$$

and

$$F_2(x) = \sin(5\pi x)$$

and define

$$\begin{aligned} y(x, t) &= F_1(-t - 3x) + F_2(t - 3x) \\ &= \sin\left(\pi\left(-t - 3x\right)\right) + \sin\left(3\pi\left(-t - 3x\right)\right) + \sin\left(5\pi\left(t - 3x\right)\right) \end{aligned}$$