

Lesson 6: More calculations

In this lesson we will use the project exercises to present additional strategies for using the CAS in SWP.

1 Project 6:

Instructions: Create a file containing the items below and their solutions. Submit .tex a version of your file to teprice@uakron.edu. The name of your files should be of the form **yourlastname06.tex** All calculations should be done using the CAS in SWP.

Exercise 1 Compute: $\lim_{h \rightarrow 0} \frac{h}{1 - \sqrt{1+h}}$ (On this exercise you should include all the details below.)

Solution 2 Consider

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{h}{1 - \sqrt{1+h}} &= \lim_{h \rightarrow 0} \left[\frac{h}{1 - \sqrt{1+h}} \left(\frac{1 + \sqrt{1+h}}{1 + \sqrt{1+h}} \right) \right] \\ &= \lim_{h \rightarrow 0} \frac{h(1 + \sqrt{1+h})}{(1 - \sqrt{1+h})(1 + \sqrt{1+h})} \\ &= \lim_{h \rightarrow 0} \frac{h(1 + \sqrt{1+h})}{1 - (\sqrt{1+h})^2} \\ &= \lim_{h \rightarrow 0} \frac{1 + \sqrt{1+h}}{-1} \\ &= \frac{1 + \sqrt{1+0}}{-1} = -2 \end{aligned}$$

Exercise 3 Compute: $\lim_{x \rightarrow 1} \left(\frac{x+2}{x-1} - \frac{x^2+5}{x^2-1} \right)$

Solution 4 *Solution:*

$$\begin{aligned}\lim_{x \rightarrow 1} \left(\frac{x+2}{x-1} - \frac{x^2+5}{x^2-1} \right) &= \lim_{x \rightarrow 1} \left(\frac{x+2}{x-1} - \frac{x^2+5}{(x-1)(x+1)} \right) \\ &= \lim_{x \rightarrow 1} \frac{(x+2)(x+1) - x^2 - 5}{(x-1)(x+1)} \\ &= \lim_{x \rightarrow 1} \frac{x^2 + 3x + 2 - x^2 - 5}{(x-1)(x+1)} \\ &= \lim_{x \rightarrow 1} \frac{3x - 3}{(x-1)(x+1)} \\ &= \lim_{x \rightarrow 1} \frac{3(x-1)}{(x-1)(x+1)} \\ &= 3 \lim_{x \rightarrow 1} \frac{1}{x+1} \\ &= 3 \lim_{x \rightarrow 1} \frac{1}{x+1} = 3 \left(\frac{1}{2} \right) = \frac{3}{2}\end{aligned}$$

Exercise 5 *Compute:* $\lim_{x \rightarrow 2^+} \frac{x+2}{x-2}$

Solution 6 *Let $t = x - 2$ and consider*

$$\begin{aligned}\lim_{x \rightarrow 2^+} \frac{x+2}{x-2} &= 4 \lim_{x \rightarrow 2^+} \frac{1}{x-2} \\ &= 4 \lim_{t \rightarrow 0^+} \frac{1}{t} \quad t = x - 2 \\ &= \infty\end{aligned}$$

Exercise 7 *Solve $3x^3 + 2x + 1 = 0$*

Exercise 8 *Solve exact*

$$\begin{aligned}a^2 - b^2 &= 5 \\ a + b &= 1\end{aligned}$$

Exercise 9 *Solve*

$$\begin{aligned}a^2 - b^2 &= 5 \\ a + c &= 1\end{aligned}$$

Exercise 10 *Combine*

$$2^a 2^b =$$

Exercise 11 *Expand*

$$\sin(a+b)$$

and

$$\sin(12\theta) =$$

Exercise 12 *Simplify and Factor*

$$\frac{a^4 - b^4}{a - b}$$

Exercise 13 π

Exercise 14 *Solve the following differential equation*

$$\frac{dy}{dx} = xy$$

Exercise 15 *Solve the following differential equation*

$$y'' + y = x^2$$

Example 16 (*Graduate - and other interested - Students*) *The wave equation*

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

has exact solution $y(t, x) = F_1(-x - at) + F_2(x - at)$ *where* F_1 *and* F_2 *are functions of a real variable. To see this note that*

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2} &= \frac{\partial}{\partial t} \frac{\partial y}{\partial t} \\ &= \frac{\partial}{\partial t} (-aF_1'(-x - at) - aF_2'(x - at)) \\ &= a^2 (F_1''(-x - at) + F_2''(x - at)) \end{aligned}$$

Likewise,

$$a^2 \frac{\partial^2 y}{\partial x^2} = a^2 (F_1''(-x - at) + F_2''(x - at))$$

and it follows that

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

As a specific example set

$$F_1(x) = \sin(\pi x) + \sin(3\pi x)$$

and

$$F_2(x) = \sin(5\pi x)$$

and define

$$\begin{aligned} y(x, t) &= F_1(-t - 3x) + F_2(t - 3x) \\ &= \sin(\pi(-t - 3x)) + \sin(3\pi(-t - 3x)) + \sin(5\pi(t - 3x)) \end{aligned}$$