

## Lesson 5: Matrices and References

### 0.1 Determinants of $2 \times 2$ matrices

Eigen values of a  $2 \times 2$  matrix are discussed in Section 0.2. Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

be a  $2 \times 2$  matrix. Then the determinant of  $A$  is the number

$$\det A = ad - bc.$$

**Example 1** *If*

$$B = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}$$

*then*

$$\det B = 18$$

We can define  $A$  to be the matrix given in the previous example and compute its determinant by evaluating

$$\det A = ad - bc$$

Sometimes the notation  $||$  is used to denote the determinant of a matrix. Consequently,

$$|A| = ad - bc$$

### 0.2 Eigenvalues of $2 \times 2$ matrices

The **eigenvalues** of a  $2 \times 2$  matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  are defined by the equation

$$\det \begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix} = 0$$

Consequently, the eigen values of  $A$  are the solutions to the polynomial equation

$$ad - bc - a\lambda - d\lambda + \lambda^2 = 0$$

The  $2 \times 2$  **identity matrix** is defined by

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The eigenvalues of the matrix  $B$  given in Example 1 are given by

$$\det(B - \lambda I) = 0$$

which reduces to

$$\lambda^2 - 10\lambda + 18 = 0$$

This last equation has solution

$$\lambda = 5 - \sqrt{7}, \sqrt{7} + 5$$

### 0.3 Project 5

Instructions: Create a file containing solutions to the items below. Your document should include items similar to those presented for the  $2 \times 2$  case. Submit .tex and .dvi versions of your file to teprice@uakron.edu. The name of your files should be of the form **yourlastname05.tex** and **yourlastname05.dvi**. All calculations should be done using the CAS in SWP.

**Exercise 2** Find a formula for the determinant of a  $3 \times 3$  matrix. *Hint:*

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

**Exercise 3** Let

$$B = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 1 & 0 \\ 4 & -3 & 2 \end{bmatrix}$$

Find  $\det B$  and the eigenvalues of  $B$ .