

Lesson 5: Matrices and References

0.1 Determinants of 2×2 matrices

Eigen values of a 2×2 matrix are discussed in Section 0.2. Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

be a 2×2 matrix. Then the determinant of A is the number

$$\det A = ad - bc.$$

Example 1 *If*

$$B = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}$$

then

$$\det B = 18$$

We can define A to be the matrix given in the previous example and compute its determinant by evaluating

$$\det A = ad - bc$$

Sometimes the notation $||$ is used to denote the determinant of a matrix. Consequently,

$$|A| = ad - bc$$

0.2 Eigenvalues of 2×2 matrices

The **eigenvalues** of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are defined by the equation

$$\det \begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix} = 0$$

Consequently, the eigen values of A are the solutions to the polynomial equation

$$ad - bc - a\lambda - d\lambda + \lambda^2 = 0$$

The 2×2 **identity matrix** is defined by

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The eigenvalues of the matrix B given in Example 1 are given by

$$\det(B - \lambda I) = 0$$

which reduces to

$$\lambda^2 - 10\lambda + 18 = 0$$

This last equation has solution

$$\lambda = 5 - \sqrt{7}, \sqrt{7} + 5$$

0.3 Project 5

Instructions: Create a file containing solutions to the items below. Your document should include items similar to those presented for the 2×2 case. Submit .tex and .dvi versions of your file to teprice@uakron.edu. The name of your files should be of the form **yourlastname05.tex** and **yourlastname05.dvi**. All calculations should be done using the CAS in SWP.

Exercise 2 Find a formula for the determinant of a 3×3 matrix. *Hint:*

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Exercise 3 Let

$$B = \begin{bmatrix} 2 & 3 & -1 \\ 5 & 1 & 0 \\ 4 & -3 & 2 \end{bmatrix}$$

Find $\det B$ and the eigenvalues of B .